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Hysteresis in a Three-Equation Model

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[1] Paper forthcoming in the *Eastern Economic Journal.* This note is based on a conversation with Engelbert Stockhammer at Kingston University in Spring 2014 where I was an EPOG (European Project on Globalization) Scholar. I've also benefited from comments by Wendy Carlin, Duncan Foley, and Amitava Dutt and two referees of the *Eastern Economic Journal.* I take full responsibility for views expressed and of course for any errors.

Abstract

Hysteresis in a Three-Equation Model
by Thomas R. Michl

JEL E11, E12, O42
Keywords: hysteresis, three-equation model, path dependence,
inflation-expectations anchoring.

This paper introduces two post-Keynesian hysteresis mechanisms into a standard textbook three-equation model. The mechanisms work through wage bargaining and price setting. Workers are assumed to change their wage aspirations when the actual wage differs from their target wage, and firms are assumed to change their mark-up norm when the actual profit share differs from their target share. These mechanisms do not themselves guarantee hysteresis. A pure inflation shock will create hysteresis even if expectations are anchored to the central bank's inflation target. After a demand shock, if inflation expectations are not anchored, these mechanisms generate persistence but not true hysteresis. But if expectations are partially (as they seem to be) or fully anchored, a demand shock will have a permanent effect on output, employment, and the real wage because in this case, the central bank is not obligated to reflate as aggressively in order to manage expectations. Hysteresis effects may explain the absence of disinflation and the fall in the wage share in the aftermath of Global Financial Crisis.

The aftermath of the Global Financial Crisis (GFC) has reawakened interest in the idea that demand shocks leave lasting effects on the level of output and employment through path dependence or hysteresis. In this paper, hysteresis refers to the property of an economic model that displays sensitivity to initial conditions so that even a temporary demand shock has permanent effects because it changes the long-run equilibrium (Carlin and Soskice, 2015, p. 564). Hysteresis-generating mechanisms that have been proposed include insider-outsider effects (Lindbeck and Snower, 1986), increases in unemployment duration resulting in skills obsolescence (Layard and Nickell, 1986), and losses in capital stock (Soskice and Carlin, 1989; Rowthorn, 1999). The mechanisms this paper considers involve changes in workers' wage aspirations and in firms' mark-up pricing norms that have been proposed by post-Keynesian economists[1] such as Engelbert Stockhammer and Peter Skott.

One motivation is that the recovery post GFC in the U.S. has been characterized by two empirical puzzles. First, the Phillips curve has virtually gone *hors de combat*, or as one survey of the landscape explains "the decline in inflation since the onset of the Great Recession has been less than many popular models of the inflation process would have predicted"(Kiley, 2015). Second, the share of wages in national income has fallen sharply, quite possibly a departure from past behavior in recoveries (Figura and Ratner, 2015, Fig. 3). The objective of the paper is to propose a simple, accessible model of hysteresis through which these two observations can be interpreted. To isolate the role of the hysteresis-generating mechanisms, we will introduce the post-Keynesian mechanisms into an otherwise conventional model taken from a leading macroeconomics text. And we will assume that complications from the zero lower boundary, the deflation trap or other non-linearities do not arise in order to present an *a fortiori* argument that hysteresis deserves

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[1] Post-Keynesian in the broad sense refers to economists who have taken their cue from Keynes's inner circle, including Joan Robinson, Nicholas Kaldor, and Roy Harrod, rather than from the North American interpreters of Keynes such as Paul Samuelson or Franco Modigliani. But there are many variants. Lavoie (2014) and Taylor (2004) provide handy guides to this terrain.

to be taken seriously.

Another motivation is that the existing literature has failed to integrate hysteresis and monetary policy rules very effectively. On the one hand, there is an extensive conventional literature on optimal monetary policy rules that was inspired by Taylor (1993). Much of this literature leads to a version of the three-equation model with a unique inflation-neutral equilibrium and without any hysteresis mechanism. On the other hand, there is a post-Keynesian literature (Skott, 2005; Stockhammer, 2008, 2011) on hysteresis mechanisms that does not discuss their interactions with monetary policy rules in much detail. This paper attempts to bridge this gap and offer some lessons for both traditions.

1 Hysteresis mechanisms

To economize on exposition we will stick to the basic assumptions, notation and even some terminology of the widely-used macroeconomics text by Wendy Carlin and David Soskice (2015) with some exceptions. Variables that are dated will carry a time subscript only when needed for disambiguation. We will suppress the t in representing variables (so z_{t-1} will be written z_{-1}). Parameters will have identifying numerical subscripts, and it will generally be clear that these are not time signatures. To further economize on notation, we assume unit labor productivity so that output, y, and employment are identical (aside from units of measure). This makes the real wage, w, equivalent to the wage share. The labor force is assumed to be constant, so that output and unemployment move inversely and we needn't consider the unemployment rate explicitly at all.

The inflation process revolves around bargaining between workers (either individually as in an efficiency wage setting or collectively through trade unions) and firms over the real wage. We will linearize the wage-setting and price-setting curves and use some notation that differs from the C/S text as follows:

$$w^{WS} = b_0 + b_1 y + B_t$$

$$w^{PS} = c_0 + c_1 y + C_t.$$

The innovation here, borrowed from Stockhammer (2008), lies in the time-dependent terms B_t and C_t, which capture the effects of *wage aspirations* and *mark-up norms*. In this paper, we will assume that initially these are normalized to zero, or $B_0 = C_0 = 0$.

We will adopt the convention that the model is in a long-run equilibrium in period 0, so that the values of important variables like output, the real wage, etc. in period 0 can be taken as benchmarks. The question we address is how the model responds to an aggregate demand shock or a change in the inflation target in period 1.

The idea is that workers form aspirations about the wage they desire based on their experience in the labor market. If employed workers receive a wage higher or lower than the wage to which they feel in some sense entitled, they revise their aspirations. We take the bargained real wage, w^{WS}, to represent this reference point. If workers receive a real wage that is higher than the wage for which they have bargained and if this gap persists over time, they will revise their wage bargain upward in subsequent periods. (Note that workers are not forming aspirations by comparing their current real wage to past wages.) We provide an example of this scenario below.

Similarly, firm managers consult the normal level of the mark-up in formulating their pricing plan. If they experience an actual mark-up that is higher or lower than the current norm, the mark-up will be revised accordingly. In this treatment the mark-up is not simply a reflection of profit-maximizing pricing under conditions of imperfect competition (i.e., an expression of the elasticity of product demand), but also reflects (as post-Keynesians have long argued) social factors such as class struggle, the growth objectives of managers, and normative behavior. Lavoie (2014, Ch. 3) provides extensive discussion of this approach.

Because the mark-up that is actually received reflects the prevailing real wage, the firm's mark-up norms can also be explained in terms of the real wage. If the prevailing real wage is less than the real wage associated with the target mark-up, w^{PS}, the actual mark-up must exceed the target mark-up and if this mismatch persists managers will be inclined to revise upward their mark-up norm in subsequent periods.[2] We will provide an example of this scenario below as well.

The rate of inflation will stabilize when the level of output reaches y_e, which we will call the *equilibrium level of output*. It is defined by consistency between the demands of workers and firms, or $w^{WS} = w^{PS}$. Notice that equilibrium output is potentially time-dependent:

$$y_e = \frac{(C_t - B_t) + (c_0 - b_0)}{b_1 - c_1}.$$

We will assume that the shift parameters are initialized to be zero so any dynamic process that leads to non-zero values for the shift parameters will change the equilibrium level of output.

These equations form the basis for the expectations- or inertia-augmented Phillips curve (take your pick) in which the lagged inflation rate affects current inflation:

$$\pi = \chi\pi^T + (1 - \chi)\pi_{-1} + \alpha(y - y_e). \tag{1}$$

The slope of the Phillips curve, α, mirrors the parameters of the wage and price setting equations in a transparent way. Here we allow for the possibility that inflation expectations are anchored by the inflation target, π^T, chosen by the central bank, with $0 \leq \chi \leq 1$ measuring the extent of anchoring. There

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[2] It may actually be less confusing to formalize this point. Recalling the unit labor productivity assumption and assuming labor is the only cost of production for simplicity, the actual real wage will be $w = 1/(1+\mu)$ where μ is the actual mark-up. The real wage reflecting the mark-up norm, μ_N, will be $w^{PS} = 1/(1+\mu_N)$. Clearly, if $w < w^{PS}$ then $\mu > \mu_N$.

is considerable evidence that over the last two decades, as inflation-targeting has become the prevailing form of monetary policy (and perhaps because of this trend), inflation expectations have become increasingly anchored and less responsive to variation in the actual inflation rate; see International Monetary Fund (2013, Ch. 3) and the citations therein.

Following Stockhammer (2008) we will assume that when the system operates away from equilibrium output the actual real wage lies somewhere between w^{WS} and w^{PS} owing to a stable lag structure. There are lags between when prices adapt to money wages and when money wages adapt to prices. The actual real wage is a weighted average of the two wage targets and obeys

$$w = \phi w^{WS} + (1 - \phi) w^{PS}$$

where $0 \leq \phi \leq 1$ is the weight. One justification for this formalization is that it can make the model consistent with the evidence that real wages are procyclical, even if, for example, the price setting real wage is countercyclical because of the behavior of the mark-up over the cycle. More importantly here, we would like to leave open the possibility that both the workers' wage aspiration and firms' mark-up norm can evolve over time.

The important point is that when the system operates away from equilibrium, both workers and firms will change their aspirations and norms. We formalize this with the following equations of motion, using the Δ operator[3] to indicate a first-difference:

$$\Delta B = \psi(w - w^{WS})$$
$$\Delta C = \sigma(w - w^{PS}).$$

The hysteresis-generating mechanisms kick in whenever the level of output deviates from the currently prevailing equilibrium and as a result, the equilibrium level of output evolves according to an equation that will play a central role in the dynamics of the system:

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[3] To be clear, $\Delta z = z_{+1} - z$.

$$y_e = \theta y_{-1} + (1-\theta) y_{e-1}. \quad (2)$$

The parameter $\theta = \phi\sigma + (1-\phi)\psi$ is a weighted sum that captures the strength of the two separate hysteresis-generating mechanisms. This parameter is the fulcrum for the hysteresis mechanism in this model. We will restrict θ to be strictly less than one for obvious reasons, so that the current equilibrium output level stays between the previous levels of equilibrium and actual output.

To visualize the hysteresis-generating mechanisms, consider a temporary negative demand shock (for example, a one-period shift in the intercept term of the IS equation) that lowers output below equilibrium. The system goes from point A on Figure 1 to point B. This will reduce the prevailing real wage depending on the magnitude of ϕ, but it will reduce the bargained real wage by more as workers are in a weaker bargaining position. (The actual real wage always lies between w^{WS} and w^{PS}.) Over time the workers' wage aspirations will tend to rise since they are receiving a wage in excess of the wage warranted by their (low) bargaining power, shifting the WS curve upward. This increase in wage aspirations occurs despite the fact that workers are receiving a real wage lower than they enjoyed in the past in the initial equilibrium; workers are comparing the actual wage to the bargained wage in forming aspirations.

The firms' mark-up norm will also tend to rise, which means that the relevant parameter in the price-setting equation (C) will tend to fall, shifting the PS curve downward. After the shock, the real wage will be lower than the real wage reflecting the mark-up norm so that the prevailing mark-up will exceed the norm as discussed above. This scenario is presented in Figure 1, with the parameter $c_1 = 0$ so that the PS curve is flat for simplicity.

Taken together these changes in the wage- and price-setting functions will tend to lower the equilibrium level of output, which is the essence of hysteresis. To determine under what conditions the mechanisms generate true hysteresis (i.e., permanent reductions in output and employment) as opposed to persistence (temporary reductions in equilibrium output), we need to incorporate them into a fully specified model. Note that we have not built in any asymmetries, such as worker resistance to reductions in their wage aspirations. Thus, the hysteresis mechanisms work in both directions (raising equilibrium output in a boom), and this is a key to understanding some of the results below.

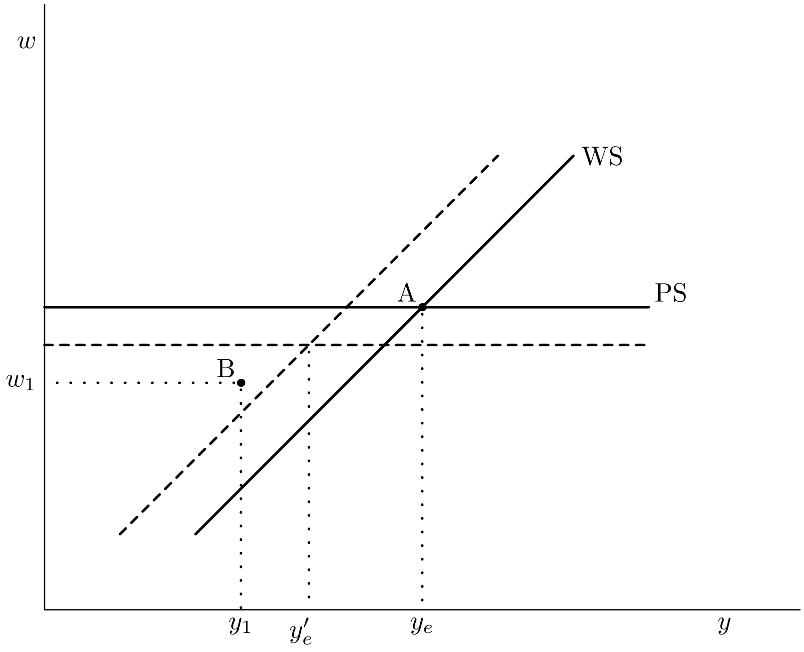

Figure 1: A prolonged slump increases the aspiration factor and the mark-up norm, shifting the WS-PS schedules and reducing the equilibrium level of output.

In Figure 1, it is clear that real wages are depressed by the hysteresis mechanisms but the model is flexible enough to accommodate other outcomes for the wage. For example, if the price-setting curve slopes downward ($c_1 < 0$), wages can rise if the price-setting curve shifts by less than the wage-setting curve. In any case, however, the hysteresis mechanisms depress equilibrium output after a negative shock.

2 Monetary policy

The central bank is assumed to minimize a quadratic loss function subject to the constraint represented by the Phillips curve. The central bank has a one-period time horizon, in the sense that it minimizes the loss function in each period without considering the whole path of adjustment after a shock.[4] The loss function is

$$L = (y - y_e)^2 + \beta(\pi - \pi^T)^2$$

where β represents the relative weight placed on missing the inflation target.[5]

In the presence of the hysteresis-generating mechanisms the central bank is assumed to observe the change in inflation and infer equilibrium output by solving (from the Phillips curve) $y_e = y + (\chi(\pi^T - \pi_{-1}) - (\pi - \pi_{-1}))/\alpha$. To achieve its objectives, the central bank sets the policy interest rate which affects aggregate demand in the next period through an IS-curve:

$$y_t = A_t - ar_{-1}.$$

The central bank is assumed to know the structure of demand aside from temporary shocks. It can determine the *stabilizing rate of interest*, r_s, that it believes will contain the inflation process by solving $r_s = (A - y_e)/a$.

From the first-order condition for this simple minimization problem, we have the *monetary rule* showing the central bank's desired outcome:

$$y = y_{e-1} - \alpha\beta(\pi - \pi^T).$$

The lagged equilibrium output level appears in the monetary rule because the central bank operates myopically on the assumption that the equilibrium

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[4] An alternative is to minimize a discounted loss function over an extended time horizon. This approach, called optimal control, has played a role in the internal discussions at the FOMC because Janet Yellen has alluded to it in public speeches. See Brayton et al. (2014).

[5] This is often erroneously described as a measure of how "hawkish" or inflation-phobic the central bank is. In fact, after a negative demand shock, a large β would be dovish since it would lead to aggressive job-creating reflation.

output it observes will in fact prevail one year forward when its policy takes effect, which generally won't be true. As a result, it will achieve its output objective but not its inflation objective. An alternative assumption is that the central bank does correctly foresee the equilibrium output level one period forward. We will briefly characterize this alternative. Both the myopia and perfect foresight assumptions turn out to lead to hysteresis, illustrating that the source of the problem is not central bank myopia. The source of the problem lies in the loss function which commits the central bank to the objective of closing the output gap no matter what the equilibrium level of output happens to be.

From the IS curve, the monetary rule and the Phillips curve, we can derive the interest rate rule that achieves the central bank's objectives, which is a form of the (John) Taylor rule.[6] The result is

$$r = r_s + h\left(\pi - \pi^T\right)$$

where the slope term in the Taylor Rule is

$$h = \frac{\alpha\beta(1-\chi)}{a(1+\alpha^2\beta)}.$$

Notice that if $\chi = 1$, the Taylor rule is reduced to simply setting the policy rate equal to the perceived stabilizing interest rate. In general, the more expectations are anchored, the less aggressive is the monetary policy response needed to achieve the central bank's objectives.

The Taylor Rule can be more usefully expressed in terms of the output level that will be achieved one period forward in response to an inflation gap. Using the IS curve we can see that

$$y = y_{e-1} - ah(\pi_{-1} - \pi^T). \tag{3}$$

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[6] The Taylor Rule is normally implemented with both the inflation gap as it is here and the output gap, $y - y_e$. As explained by Carlin and Soskice (2006, pp 153-157), this form emerges when there is a time lag between output and inflation in the model specification. The model here includes no lag between output and inflation.

In this form, it is clear how the observed equilibrium output level affects output through central bank myopia. We will briefly consider a model with foresight in which the lagged equilibrium output level is replaced by the current level of equilibrium output.

3 Sacrifice ratio

The cumulative loss of output required in order to reduce the inflation rate from π_1 to π^T can be measured by $\Sigma_1^\infty(y_{+1} - y_{e+1})$. (Recall that we are following the convention that a shock occurs in period 1.) The ratio between this loss of output and the reduction in inflation is called the *sacrifice ratio* and defined by

$$R = \frac{\sum_1^\infty (y_{+1} - y_{e+1})}{\pi^T - \pi_1}.$$

(If inflation is initially below target, of course, the sacrifice ratio measures the amount of excess output that must be generated in order to reflate the economy.)

The sacrifice ratio in the strictest sense should be computed starting from the benchmark equilibrium level of output in order to measure the pure cost of disinflation. Let us label this narrow sacrifice ratio R_π.

In the presence of hysteresis-generating mechanisms, any initial output gap will disturb the equilibrium level of output and that will create additional cumulative output effects. Let us call the ratio between these cumulative output gaps and initial output gap R_y.[7] With these definitions we have a complete breakdown of the total cumulative output gaps starting from any arbitrary initial condition, (y_1, π_1):

$$R(\pi^T - \pi_1) = R_\pi(\pi^T - \pi_1) + R_y(y_1 - y_{e0}).$$

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[7] To be precise define $R_y = \Sigma_1^\infty(y_{+1} - y_{e+1})/(y_1 - y_{e0})$.

The first step to deriving the broad sacrifice ratio is to use the Phillips curve cumulated forward:

$$\sum_{1}^{\infty} \Delta\pi = \chi(\pi^T - \pi_1) + \chi \sum_{1}^{\infty} (\pi^T - \pi_{+1}) + \alpha \sum_{1}^{\infty} (y_{+1} - y_{e+1}).$$

In the special case with no anchoring ($\chi = 0$), this equation simplifies[8] immediately to the sacrifice ratio $R = R_\pi = 1/\alpha$. We will use this fact to clarify the behavior of the system in the presence of hysteresis-generating mechanisms.

With anchoring, we can use the Taylor rule and equation 2 to derive the pure sacrifice ratio, R_π, and its output-gap counterpart, R_y:

$$R_\pi = \frac{ah}{\alpha a h + (1+\theta)\chi}$$

$$R_y = \frac{-\alpha\chi\theta}{\alpha a h + (1+\theta)\chi}.$$

With these results, it is clear that the overall sacrifice ratio depends on the initial conditions:

$$R = R_\pi + R_y \left(\frac{y_1 - y_{e0}}{\pi^T - \pi_1} \right).$$

This expression will be useful in characterizing the dynamic behavior of the model.

With no anchoring the sacrifice ratio as we have seen will be $1/\alpha$. With full anchoring the pure sacrifice ratio, R_π, collapses to zero because no output gap is needed in order to manage inflation expectations (recall that in this case $h = 0$). The overall sacrifice ratio also declines, although not to zero.[9] Anchoring makes the inflation process to some degree self-correcting and reduces the need to rely on an output gap in order to manage inflation

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[8] Note that $\Sigma_1^\infty \Delta\pi = \pi^T - \pi_1$.

[9] This is due to the behavior of R_y which takes the value $-\alpha\theta/(1+\theta)$ when $\chi = 1$.

expectations. The inverse relationship between the sacrifice ratio and the degree of anchoring will prove invaluable for interpreting the dynamics of the model.

It is of some general interest that under an intermediate degree of anchoring the weight the central bank places on hitting its inflation target enters into the pure sacrifice ratio, R_π. Indeed, the more weight the central bank puts on hitting its inflation target, the more sacrifice it demands during a disinflation and the larger is the sacrifice ratio. The intuition here is that anchoring makes the inflation process to some degree self-correcting and a more impatient central bank is forgoing some of this painless disinflation.

4 Three equation model plus one

Taken together, the Phillips curve, the IS equation, and the Taylor rule are often called the three-equation model. Actually, this system can be solved for the path of output and inflation using only the monetary rule and the Phillips curve without reference to the IS curve, since that just shows the means by which the central bank achieves its objectives. When we combine the hysteresis mechanisms with those two equations, we have a complete dynamic system comprising equations 1, 2, and 3.

These three equations form a first-order 3X3 system of difference equations that can be compactly represented in matrix form:

$$\mathbf{y} = \mathbf{A}\mathbf{y_{-1}} + \mathbf{b} \tag{4}$$

where the column vector $\mathbf{y}= [\pi, y, y_e]'$.[10] The matrix $\mathbf{A}$ and column vector $\mathbf{b}$ are

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[10] An alternative strategy would use the Phillips curve, the IS equation, and the Taylor Rule to solve for inflation, the interest rate, output and equilibrium output.

$$\mathbf{A} = \begin{pmatrix} 1-\chi-\alpha a h & -\alpha\theta & \alpha\theta \\ -ah & 0 & 1 \\ 0 & \theta & (1-\theta) \end{pmatrix}$$

$$\mathbf{b} = \begin{pmatrix} (\chi+\alpha a h)\pi^T \\ ah\pi^T \\ 0 \end{pmatrix}$$

This kind of linear dynamical system presents few mathematical challenges.[11] For it to describe a well-behaved economic model, the matrix **A** must satisfy the stability condition that its eigenvalues (roots) lie weakly within the unit circle. The stability condition is fairly easily met in numerical simulations and is characterized formally in an appendix.

The most important feature of equation 4 is that it will have one eigenvalue (sometimes called the characteristic or latent root) exactly equal to one, as is apparent from the characteristic equation for the system (using λ to represent the root):

$$(1-\lambda)\left(\lambda^2 + (\chi + \alpha a h - (1-\theta))\lambda - \theta(1-\chi)\right) = 0.$$

The presence of a unit root explains why this model displays hysteresis or sensitivity to initial conditions.[12] Without the hysteresis-generating mechanisms ($\theta = 0$), a system like this lacks a unit root and converges (assuming it is stable) on a point, which in this case would be (π^T, y_e) where y_e is a parameter.[13] But in the presence of a unit root, the system has multiple equilibria and converges on a line or plane assuming the other two roots lie

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[11] For more details on the formal properties of this type of system, see Elaydi (2005, Ch. 3) or Gandolfo (1997, Ch. 18).

[12] Another feature of the characteristic equation is that its discriminant is strictly positive which rules out complex roots that would generate cyclic behavior.

[13] Formally, we can solve for the steady state where $\mathbf{y} = \mathbf{y_{-1}} = \mathbf{y}^* = (\mathbf{I}-\mathbf{A})^{-\mathbf{1}}\mathbf{b}$. This is sometimes called the particular solution to the system. In the presence of a unit root, $(\mathbf{I}-\mathbf{A})$ is singular and its inverse is not defined.

within the unit circle. In this case, the equilibrium can lie anywhere along the line $\pi = \pi^T$. The ultimate value of y_e depends on the initial conditions.[14]

Assuming we have a stable system that converges, the next order of business is studying its fixed-point solution, where $\mathbf{y} = \mathbf{y_{-1}} = \mathbf{y}^*$, and exploring its properties after the original equilibrium has been disturbed. Since we have adopted the convention that a full equilibrium prevails in period 0, we can take $\mathbf{y_0}= [\pi_0^T, y_0, y_{e0}]'$ to be the benchmark against which we compare the ultimate steady state equilibrium achieved when the system converges, $\mathbf{y}^*= [\pi^*, y^*, y_e^*]'$. We will also be interested in the behavior of the interest rate and the real wage which are auxiliary variables to this system that can always be calculated from $\mathbf{y}^*$. We consider two kinds of shocks–demand shocks and inflation shocks–that perturb the system in period 1. We will consider the position in period 1 to constitute the initial conditions.

5 The role of initial conditions

Before tackling the specific shocks that interest economists, let us consider the general question of what initial conditions lead to positive hysteresis effects, negative hysteresis effects or no hysteresis effects. Formally, we seek the initial conditions that make $y^* > y_{e0}$, $y^* < y_{e0}$ or $y^* = y_{e0}$ or equivalently that make $\Sigma_1^\infty \Delta y_e > 0, < 0$, or $= 0$. From equation 2 we know that $\Sigma_1^\infty \Delta y_e = \theta \Sigma_1^\infty (y - y_e)$. Taken together with what we learned about the sacrifice ratio we can see that

$$\theta \sum_1^\infty (y - y_e) = \theta(y_1 - y_{e0}) + \theta R(\pi^T - \pi_1).$$

The first term on the right-hand side represents the hysteresis effect from the initial output gap while the second term represents the effect from the total cumulative output gap created by the policy response to the initial

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[14] For further discussion of the role played by unit roots (or zero roots in continuous time models) see Amable et al. (1993) or Dutt (1997).

conditions. The boundary between positive and negative hysteresis effects will be given by setting $\theta(y_1 - y_{e0}) + \theta R(\pi^T - \pi_1) = 0$. Using what we learned about the breakdown of the sacrifice ratio we arrive at the condition for no hysteresis effects:

$$\pi_1 = \pi^T + \frac{1 + R_y}{R_\pi}(y_1 - y_{e0}). \tag{5}$$

This equation forms an important boundary. Initial conditions that lie above (below) this line on the (y, π) plane will lead to negative (positive) hysteresis effects. We use this fact to characterize the effects of demand shocks and inflation shocks.

6 Demand shocks

We will consider temporary demand shocks that decrement the autonomous spending term, A, in the IS equation for one period and then disappear. We could extend the model to permanent shocks that affect A for all future periods but as long as the central bank realizes that the structure of demand (the IS curve) has changed and makes allowances through its policy rule, the outcome will not be much different as far as our object of interest–hysteresis–is concerned.[15] The expositional advantage of focusing on temporary shocks is that any change in the sustainable rate of interest will reflect hysteresis effects *tout court*. This treatment is equivalent to initializing the system in period 1 to a point on the Phillips curve below its benchmark position in period 0.

This raises the issue of the central bank's response to the policy shock. We know that the hysteresis mechanism will alter the equilibrium level of

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[15] Both cases pose an initial condition problem. In practice, distinguishing between permanent and temporary shocks raises hard questions about how the central bank knows what the structure of demand and sustainable rate are in the first place. I have chosen to avoid this complication in order to focus attention on the pure interaction of hysteresis and policy, not because permanent shocks are unimportant.

output and change the dynamics of the inflation process in period 2 after a demand shock in period 1. Our assumption is that in period 2 the central bank, perhaps through its staff economists, will observe the new equilibrium level of output and make the appropriate change in its policy rule. In each subsequent period, the central bank repeats this learning process. In this way, the central bank will modify its belief about the stabilizing rate of interest, r_s, in each period. We are assuming that the central bank treats all changes in the observed equilibrium level of output as supply-driven, which is consistent with the prevalent belief among macroeconomists (sometimes called the "accelerationist hypothesis") that the natural rate of unemployment reflects structural features of labor and product market institutions that cannot be altered by demand management.

We will briefly relax this assumption below and endow the central bank with enough foresight to predict the equilibrium level of output one period forward in order to put to rest any suspicion that myopia is somehow responsible for the results. The maintained hypothesis in this paper is that the central bank takes its objective to be closing the output gap at whatever equilibrium level of output emerges. Our question is then posed with some precision: if the central bank operates in this fashion, will its behavior translate potential path dependence into actual path dependence.

These are strong assumptions but we do need to incorporate some adjustment in the policy rule in order to accommodate potential hysteresis effects. If we were to allow the central bank to choose the wrong value for r_s indefinitely, it is well known that in this kind of model the system (assuming it is stable) will converge on an inflation rate that differs from the target.[16] In the absence of any hysteresis mechanism, this would not be too problematic although it clearly is unsatisfactory. But if the central bank makes this error in the presence of hysteresis mechanisms, it could compound the problem by also creating hysteresis effects, effectively the iatrogenic effects of bad pol-

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[16] The inflation rate will deviate by $\pi - \pi^T = (\tilde{r} - r_s)/h$ where $\tilde{r}$ is the perceived stabilizing rate of interest.

icy. This may be important in practice–witness the tendency for inflation to fall below target in the US and Eurozone, which might be a symptom of an inappropriate choice for the stabilizing rate. But it is a distraction from the question we have posed above.

6.1 With no anchoring

We are now prepared to state our first result. In the absence of anchoring ($\chi = 0$), the system does not display hysteresis after a demand shock. To be more precise, it returns to the benchmark equilibrium:

$$\mathbf{y}^* = \mathbf{y_0}.$$

This is a remarkable result and it deserves some explanation. A negative demand shock in this type of model necessitates that the central bank respond by reflating the system in subsequent periods. Indeed, as we have seen the sacrifice ratio with no anchoring is invariant to the timing of the reflation (i.e., it is independent of β), and the central bank is going to have to create enough positive ($y > y_e$) point-years of output gap to compensate for the negative gap created by the shock. This is a type of conservation principle that draws on our earlier conclusion that with no anchoring the sacrifice ratio is $1/\alpha$.

To show this more formally we refer to the boundary condition described by equation 5. With $\chi = 0$ this reduces to

$$\pi_1 = \pi^T + \alpha(y_1 - y_{e0})$$

which of course is the Phillips curve. In the absence of anchoring a demand shock that moves the system along the prevailing Phillips curve will have no hysteresis effect in the long run. To visualize this, consult the left panel of Figure 4 below.

The key point is that starting from a point on the Phillips curve, the central bank restores the same number of point-years of excess output that

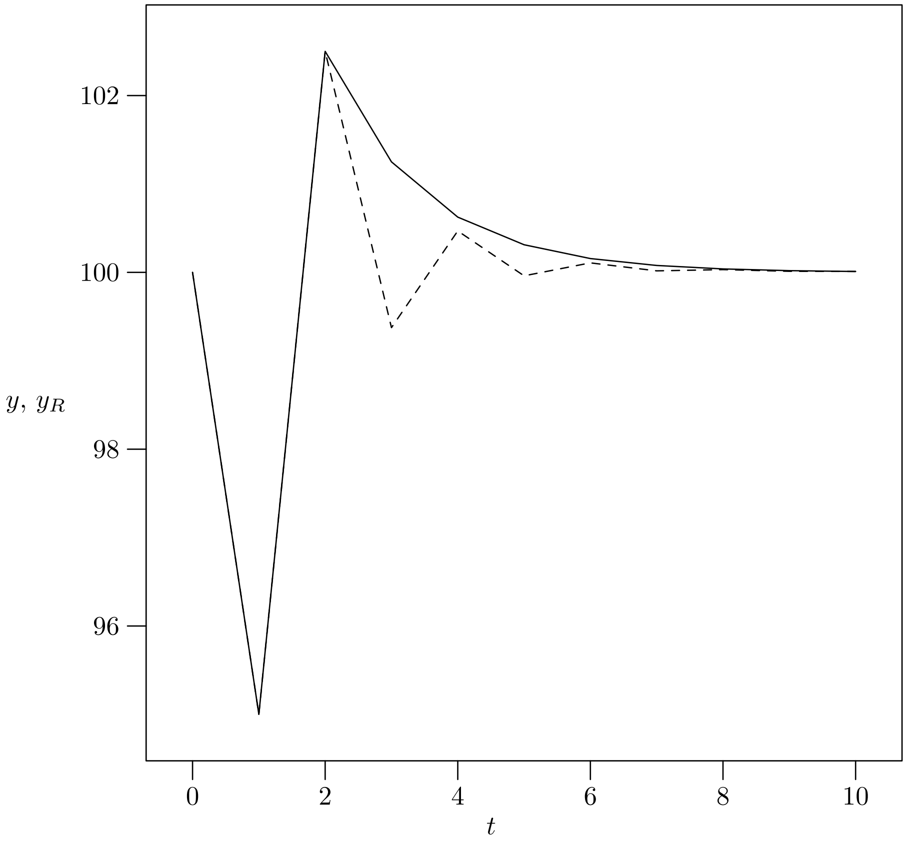

Figure 2: A demand shock of -5 in period 1 with no anchoring ($\chi = 0$) results in persistence. Output in a reference scenario (y_R) with no hysteresis mechanism is the solid line; output in the test case is dashed. See Appendix for parameter values.

were withdrawn by the shock. Given the adaptive or inertial character of the inflation process, the central bank is obliged to reflate in order to manage expectations and return inflation to its target. It is this reflation that sends the hysteresis mechanisms into reverse, ultimately neutralizing all the damage done to y_e by the shock. This is one argument in favor of keeping inflation on target.

However, it would be a mistake to paint too rosy a picture of the hysteresis mechanisms because their effect is to create persistently lower output and employment during the period of adjustment. This can be seen in Figure

2, which shows the impulse response function (IRF) for output with and without the hysteresis mechanisms in place in a model solved recursively and calibrated for stability (parameter values are given in an appendix). Call these the test and reference scenarios; both scenarios involve a -5 per cent demand shock. The reference IRF shows the response to the shock when $\psi = \sigma = 0$ and the hysteresis process is shut down. It is clear that the central bank has responded in the test scenario with less stimulus. For an unemployed worker in the test scenario who might have been employed in the reference scenario, the distinction between persistence and hysteresis might be regarded as somewhat academic.

6.2 With anchoring

Our second result is that in the presence of inflation-expectation anchoring ($\chi > 0$), the system does display hysteresis after a demand shock. To be more precise, it does not return to the benchmark equilibrium so that

$$\mathbf{y}^* \neq \mathbf{y_0}.$$

In particular, for a negative demand shock, we see that $\pi^* = \pi^T$, $r^* > r_{s0}$, and $y^* < y_{e0}$. To convey a sense of the full dynamics, Figure 3 presents the IRFs for the key variables of interest–output, equilibrium output, the real wage, the interest rate, and inflation–with an intermediate degree of anchoring.[17]

This is also a remarkable result. Expectations anchoring is generally regarded as a good thing since it relieves the central bank of the necessity to manage expectations through demand management. Yet this is precisely the problem in the presence of hysteresis mechanisms. Because the central bank is not obliged to manage expectations as aggressively, it does less salubrious

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[17] Fully anchored inflation expectations, as we showed, are reflected in a policy of setting the interest rate at its stabilizing level. But the rate of interest acts with a lag, so until it converges on the steady state the central bank will find itself missing its desired inflation rate because it does not perceive the forward change in equilibrium output.

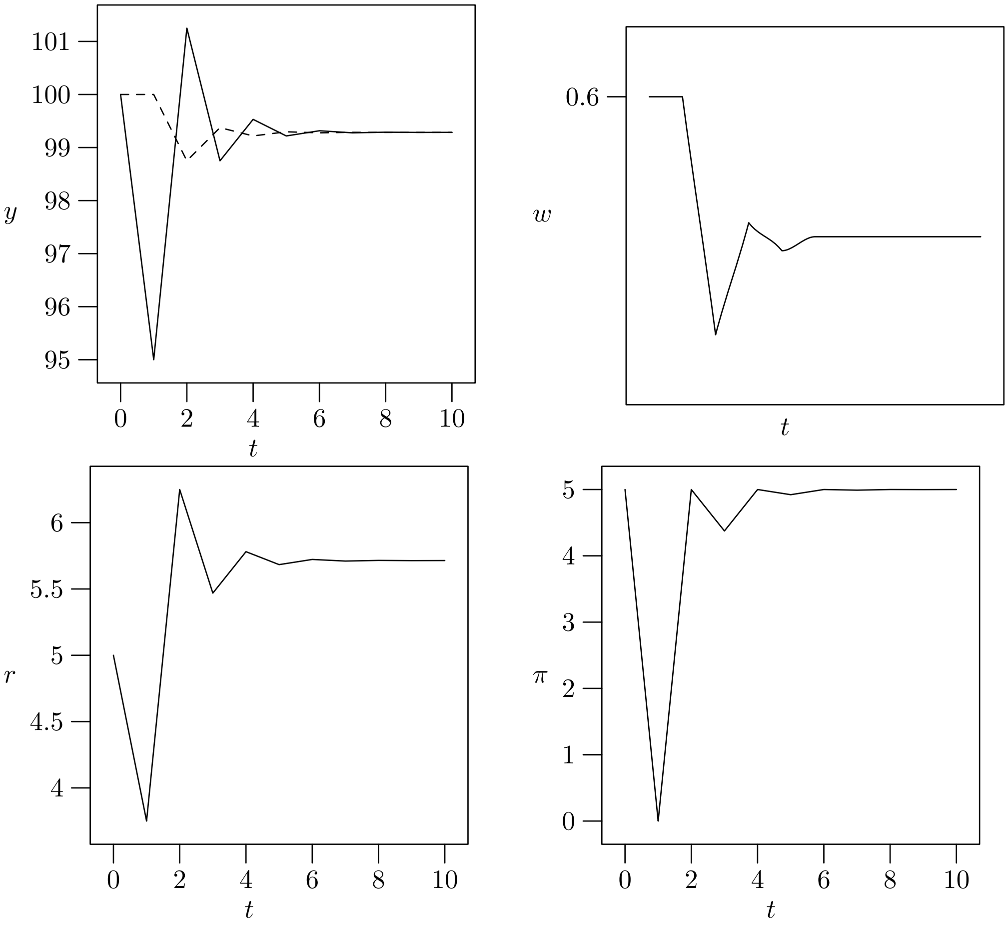

Figure 3: A demand shock of -5 in period 1 with intermediate anchoring ($\chi = 0.5$) results in hysteresis. The equilibrium level of output (y_e) is dashed. For parameter values, see Appendix.

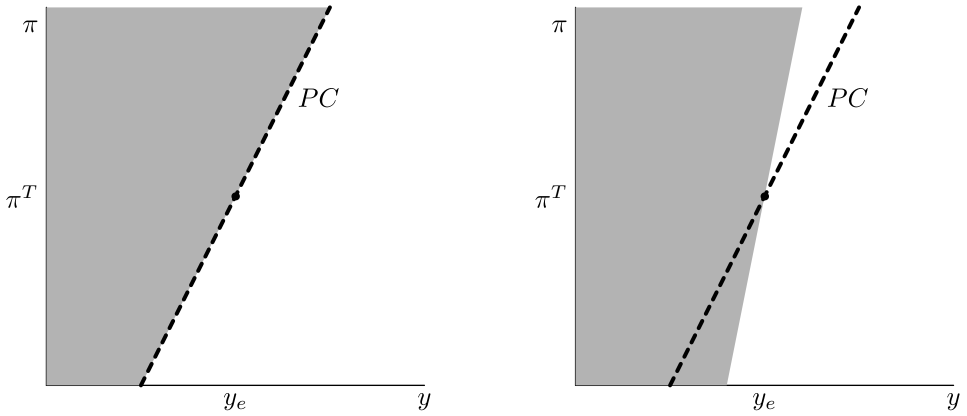

Figure 4: Left: With no anchoring, the Phillips Curve (dashed line labelled PC) forms the boundary between positive and negative hysteresis effects. Shading shows the region of negative effects. Right: With partial anchoring, the boundary rotates around (y_e, π^T) and contains the Phillips Curve. With full anchoring (not shown) the boundary becomes a vertical line $y = y_e$.

reflating and consequently fails to reverse the hysteresis process fully. Put another way, because anchoring lowers the sacrifice ratio as we saw earlier, the central bank does not need to fully replace demand that has been withdrawn by the demand shock (as it did with no anchoring). The demand shock inflicts permanent damage on output and employment as a result.[18]

To demonstrate this more formally, we again refer to equation 5. As we showed earlier, an increase in the degree of expectations anchoring will reduce the sacrifice ratio. This effectively rotates the boundary condition counterclockwise around the pivot at (y_{e0}, π^T). Thus, a point on the Phillips curve with $y_1 < y_{e0}$ will now lie above the boundary condition, which will send it to a lower long-run level of equilibrium output. To visualize this, consult the right panel of Figure 4.

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[18] In the simulation in Figure 3 with $\chi = 0.5$ the equilibrium level of output falls from 100 to 99.3. With full anchoring, it falls to 99.0.

7 Disinflation

The final shock to consider is a policy-led disinflation which we can model by assuming that the inflation target is lowered in period 1. This is equivalent to a pure inflation shock in period 1 that raises π above π^T with no change in output, or alternatively to an initialization that puts the system above the Phillips curve prevailing in period 1. The concept of hysteresis was, of course, introduced into macroeconomics in the 1980s to describe the possible negative consequences of the widespread disinflations associated with Reagan, Volcker, Thatcher, and particularly with various European governments and central banks; Ball (1999) is still worth consulting for evidence consistent with this explanation.

In the context of this model, we are effectively extending the concept of expectations anchoring to cover central bank credibility. The change in target is assumed to be credible to the same extent that the original target was credible.

Here we can state our third major result. Unless there is complete inflation anchoring ($\chi = 1$), in response to a disinflationary policy the system will display hysteresis and will fail to return to the benchmark equilibrium. Apart from the inequality $\pi* \neq \pi_0$ (which is the policy goal), all the other elements in the steady state solution vector will obey

$$\mathbf{y}^* \neq \mathbf{y_0}.$$

Again, we find that $r^* > r_{s0}$ and $y^* < y_{e0}$. More formally with reference to equation 5, it is obvious by construction that we have chosen initial conditions that lie above the boundary condition, thus guaranteeing the result. Note that when $\chi = 1$ the boundary condition becomes the vertical line $y = y_{e0}$ so a pure inflation shock has no hysteretic effect.

Figure 4 shows the IRFs for the major variables for a scenario with an intermediate degree of anchoring.

Disinflation acts like friction in a physical system; it drains energy out of

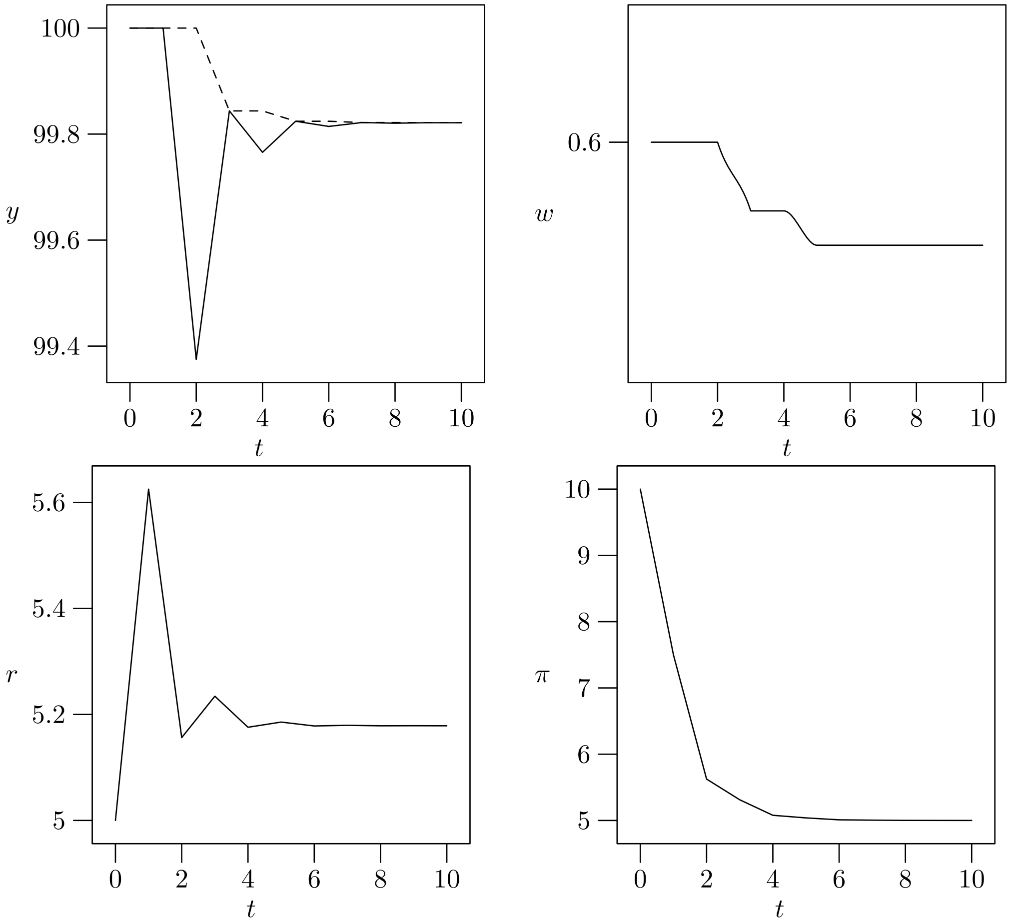

Figure 5: A disinflation with intermediate inflation-expectations anchoring ($\chi = .5$) results in hysteresis. The initial inflation target is 10 and the new target beginning in period 1 is 5 per cent per year. Equilibrium output (y_e) is dashed. For parameter values, see Appendix.

the system. Unlike a negative demand shock, which requires some reflation on the part of the central bank, disinflation leads to no policies that reverse the hysteresis process. As a result, the sacrifice ratio is somewhat poorly defined.

The exception occurs when there is complete anchoring. This is a case of the painless disinflation under perfect foresight or rational expectations that was once promised by New Classical macroeconomists. With respect to disinflation, then, anchoring is a good thing. However, in the post Global Financial Crisis world, raising the inflation target has become a contentious issue. Raising the inflation target in this model produces positive hysteresis effects. With respect to reflation, anchoring obstructs these positive effects, which include both lower unemployment and possibly higher wages.

8 Perfect foresight

Allowing for some foresight about the one-period forward equilibrium level of output on the part of the central bank changes the formal structure of the model marginally but does not eliminate the results with respect to hysteresis in any of the examples above. Further details are left for the appendix. The most significant substantive difference is that foresight actually amplifies any hysteresis effects. With foresight, the sacrifice ratio will be

$$R = \frac{\alpha\beta(1-\chi)}{\chi + \alpha^2\beta}$$

which is larger than the sacrifice ratio in the myopic model.[19] This implies that after a demand shock with an intermediate degree of anchoring the central bank will not reflate as aggressively since it recognizes that the equilibrium level of output will decline through the hysteresis-generating mecha-

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[19] This is true for two reasons. First, this ratio is larger than the pure sacrifice ratio, R_π, which can be seen by substituting for h to make the two expressions comparable. The myopic $R_\pi = \alpha\beta(1-\chi)/((\chi + \alpha^2\beta) + \theta\chi(1+\alpha^2\beta))$. Second, in the myopia model, the output effect, R_y, reduces the overall sacrifice ratio in the myopic model.

nisms and thus less stimulus will be needed to achieve its inflation objectives. The change in equilibrium output associated with any initial shock (including a mix of inflation and output shocks) will be characterized by the following equation in the perfect foresight model:

$$\sum_{1}^{\infty} \Delta y_e = \theta(y_1 - y_{e0}) + \theta R(\pi^T - \pi_1).$$

Thus, we can see than any initial demand shock or inflation shock will leave a larger footprint with central bank foresight than with central bank myopia.

9 Conclusion

This paper has combined a standard textbook 3-equation model with a post-Keynesian hysteresis-generating mechanism. The pay-off from this pluralistic approach is that it generates insights for the textbook model with important policy implications, but it also offers some lessons for post-Keynesian theory.

The presence of the hysteresis-generating mechanisms proposed in this paper does not guarantee that this class of three-equation models always displays path dependence or hysteresis. In the absence of any inflation-expectations anchoring, we found that the model displays persistence but not true hysteresis after a demand shock. The aggressive reflation required after a demand shock reversed the damage and returned the labor and product markets to their original condition. In this sense, the post-Keynesian school may be overly pessimistic about the prevalence of path dependencies. At the least, it is important to recognize that identifying the presence of a generating mechanism, post-Keynesian or otherwise, does not establish the presence of hysteresis.

However, expectations anchoring (for which there seems to be persuasive evidence) attenuates this salutary policy response to a demand shock, so that the temporary demand shock does permanent damage in the form of a reduced equilibrium level of output, employment, and possibly real wages. The "missing disinflation" (Kiley, 2015) and the declining labor share of income during the recovery from the GFC may be symptoms of the form of hysteresis theorized in this paper. Significantly, their appearance during a period with a historically high level of expectations anchoring lends some credibility to the model. This does not, of course, exclude other explanations, such as the downward stickiness of money wages (Akerlof et al., 1996) that is often proposed to explain the recent behavior of price inflation. Nonetheless, this result calls into question the conventional wisdom that expectations anchoring is an unambiguously desirable outcome for central bankers.

These results strengthen the case that hysteresis should be taken seriously since they were derived by incorporating hysteresis-generating mechanisms lacking in any asymmetric features into an otherwise conventional three-equation model with no non-linearities. Both non-linearities and asymmetries might prejudice the case for path dependence, and in fact make good follow-up questions for future research. It might also be useful to extend the model to address permanent demand shocks (which would require specifying how the central bank determines what the sustainable interest rate is), or to incorporate more complex and more realistic lag structures for the hysteresis mechanisms or for the Phillips curve. Moreover, it would be appropriate to ask what the central bank should do in the presence of the hysteresis-generating mechanisms which would require rethinking the standard loss function. Finally, the model can potentially generate testable hypotheses and an empirical research program.

The importance of work on this topic cannot be understated. Hysteresis complicates the central bank's policy decision because the losses of jobs and possibly real wages from poor choices are permanent rather than temporary as they would be under the accelerationist hypothesis.

Mathematical Appendix

This appendix provides some additional mathematical detail about the two models and the numerical values used in the simulations.

Stability in the basic model

The eigenvalues or characteristic roots of the basic system, equation 4, are $(1, \lambda_2, \lambda_3)$ where

$$\lambda_{2,3} = \frac{1}{2}\left(-(\chi + \alpha a h - (1-\theta) \pm \sqrt{\Delta}\right)$$

and $\Delta = (\chi + \alpha a h - (1-\theta))^2 + 4\theta(1-\chi)$ is the discriminant. Note that with the parameter restrictions in the model, $\Delta > 0$ so we know the roots are all real numbers. To characterize stability, we replace h with its definition in the paper, solve for $|\lambda_{2,3}| < 1$ and arrive at this stability condition:

$$\theta < \frac{2 - \chi + \alpha^2\beta}{(2-\chi)(1+\alpha^2\beta)}.$$

For example, with the numerical examples in the paper and no anchoring, stability requires that $\theta < 3/4$. With full anchoring ($\chi = 1$) stability is guaranteed since $\theta < 1$. Higher values of β (the central banks relative preference for closing its inflation target) require a smaller value of θ to maintain stability.

Perfect Foresight Model

The system with central bank foresight of the one-period ahead equilibrium level of output has the same form with slightly different coefficients. Let us call the matrix and vector **B** and **c** to avoid confusion. We have

$$\mathbf{B} = \begin{pmatrix} (ah)/(\alpha\beta) & 0 & 0 \\ -ah & \theta & (1-\theta) \\ 0 & \theta & (1-\theta) \end{pmatrix}$$

$$\mathbf{c} = \begin{pmatrix} \frac{(\chi+\alpha^2\beta)\pi^T}{1+\alpha^2\beta} \\ ah\pi^T \\ 0 \end{pmatrix}$$

There is an obvious linear dependence between the last two columns indicating that the matrix is less than full rank and has only two non-zero roots. The roots are $(1, (ah)/(\alpha\beta), 0)$. Expanding $(ah)/(\alpha\beta) < 1$ gives the stability condition

$$\frac{\alpha\beta(1-\chi)}{1+\alpha^2\beta} < 1$$

which is satisfied for any permissable parameter values.

Calibration

$$\begin{aligned} b_0 &= 0.5 & b_1 &= 0.001 \\ c_0 &= 0.7 & c_1 &= -0.001 \\ \phi &= 0.5 & \pi^T &= 5 \\ \psi &= 0.2 & \sigma &= 0.3 \\ \alpha &= 1 & \beta &= 1 \\ A &= 105 & a &= 1 \\ y_{e0} &= 100 & r_{s0} &= 5 \end{aligned}$$

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