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Abstract

This paper explores the dynamic behavior of a Romer-style endogenous growth model, analyzing how changes in tax rates affect government revenue in the short run and the long run. I show that in this environment lowering taxes on financial income is unlikely to stimulate tax revenue in the long run and has modest effects on the tax base, contrary to some other studies of the dynamic response of revenue to tax rates. Calibrations of the model that suggest Laffer curve effects can be substantial require implausibly low values for the elasticity of substitution between varieties of intermediate goods. For more plausible parameter values, I find that around 20% of a tax cut would be self-financing due to an expansion in the tax base.

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1. Introduction

Congressional Budget Office (2002) projects large fiscal deficits for most of the 21st century in the United States due to expansions in spending on Social Security, Medicare, and Medicaid. The demographic trends driving much of these spending expansions are common to many developed countries as the post World War II baby boom generations age. As a consequence, similar fiscal concerns prevail in other developed economies also (Carone and Costello (2006); Faruqee and Mühleisen (2003)). Increasing tax revenues is one way to close the budget deficit. What will be the effects if large tax increases are legislated?

This paper explores the consequences of changes in financial income tax rates in a Romer (1990)-style endogenous growth model. In doing so, it adds to the growing literature on dynamic revenue estimation and illustrates some of the dynamic behavior of ideas-based growth models. Using a growth model allows for the change in tax policy to have different effects in the short run and the long run. An endogenous growth model allows some of these effects to operate through advances in productivity. One might think that higher tax rates reduce revenue both because they reduce the incentive to accumulate physical capital and because they reduce the incentive to innovate.

In fact, the Laffer curve phenomenon is absent in the Romer model I calibrate for most plausible parameter values. A key parameter is θ, the elasticity of substitution between varieties of intermediate goods. For low values of θ, the elasticity of output with respect to the stock of ideas is very high, so incentives to innovate have important effects on output and the tax base. Low values of θ generate Laffer curve effects at low tax rates. However, such low values of θ imply very large markups in the model, in contrast to observed markups.

Elasticities in the research and development production function also matter for the response of the economy to changes in tax rates. In particular, if the returns to scale in labor are low, Laffer curve effects are less likely. With the way the allocation of resources is decentralized, low returns to scale appear as externalities ("stepping on toes"). Workers respond to lower tax rates by switching into the R&D sector, but in doing so they hamper the productivity of other workers. This curbs the response of the economy to a reduction in tax rates.

Traditional approaches to estimating the revenue effects of tax rate changes focused on static behavioral responses – essentially the short-run response of labor and capital income to a change in the tax rate.[1] Fullerton (1982) argues that Laffer curve effects (higher government revenue at lower tax rates) are unlikely for labor income taxes due to low labor supply elasticities. (Malcomson (1986) studies the same question and emphasizes the relevance of general equilibrium effects.)

In spite of skepticism about the short-run revenue enhancing effects of tax cuts in the 1980s, recent studies have considered the possibility that the long-run effect of a tax cut is to expand the government's tax collection. Economic research has generally been skeptical of large short-run behavioral responses to tax rate changes. By contrast, more economists believe that long-run responses of labor supply and especially of capital supply may be large, potentially justifying lower tax rates.[2] As Mankiw and Weinzierl (2006) point out in the context of the Ramsey model, the accumulation of capital means that a lower tax rate on capital will ultimately increase the tax base, limiting the long-run reduction in revenues from a tax rate reduction. Auerbach (1996) gives a general presentation of issues related to dynamic scoring. See also Auerbach and Kotlikoff (1987) who study a wide range of issues related to dynamic aspects of fiscal policy.

A number of other studies have considered the dynamic effects of taxes on government revenue in endogenous growth models. Those who have used AK models to explore the effects of taxes include Barro and Sala-i Martin (1992); Stokey and Rebelo (1995); Agell and Persson (2001); Ireland (1994); Bruce and Turnovsky (1999). The results of the AK model are fairly straightforward to de-

[1]This issue gained prominence in the early 1980s when some claimed that the United States had tax rates so high that lower tax rates would *increase* tax revenue. This hypothetical situation was known as being on the wrong side of the Laffer Curve.

[2]A notable counterexample is Goolsbee (2000) who argues that a reduction in high-income tax rates had large short-run effects but small long-run effects.

velop. Production is proportional to capital, $Y = AK$, though individuals may perceive this production function to be $Y = \tilde{A}K^{\alpha}L^{1-\alpha}$. Absent depreciation, the real interest rate is $r = \alpha A$. The growth rate of the economy is determined from the consumption Euler equation: $\dot{C}/C = \sigma((1-\tau_v)\alpha A - \rho)$, where σ is the intertemporal elasticity of substitution, ρ is the discount rate, and τ_v is the tax rate on capital income. As the tax rate on capital income τ_v falls the steady-state growth rate of the economy increases. Tax cuts therefore tradeoff current revenue losses for future revenue gains.

In the AK model above, the optimal growth rate exceeds the growth rate in the decentralized allocation. As such, the appropriate policy is to subsidize capital income, rather than taxing it.[3] It is natural to think that lower taxes (at least when such taxes are positive) would expand the tax base. In the model I discuss, there are several distortions that make it uncertain, a priori, whether a decentralized allocation will result in too much or too little investment in research and development.

Others have used models in which growth is driven by the accumulation of human capital, as in Lucas (1988). Examples include Novales and Ruiz (2002); Pecorino (1995); De Hek (2006); Milesi-Ferretti (1998); Milesi-Ferretti and Roubini (1998); Hendricks (1999). While some find that lower tax rates provide extensive stimulus to the economy, others report more modest responses. For example, Hendricks (1999) presents a life-cycle model with human capital accumulation. In his model human capital accumulation drives the economy in the long run, but lower tax rates do not generate large increases in the scale of the economy.

Another strand of the literature on the effects of taxation discusses the uses of government revenue. For example, Jones et al. (1993, 1997) modify the classic Chamley (1986) and Judd (1985) result that the optimal tax rate on capital income is zero. In their model the government uses tax revenue to provide productive public goods. Cutting taxes means having to cut public services, which

[3]When there is no population growth or depreciation, the optimal subsidy to capital income, financed by lump sum taxes, would be $(1-\alpha)/\alpha$.

may reduce output. Ferede (2008) follows a similar approach. In these papers, a reduction in the tax rate may not be followed by large increases in the tax base since the government has to reduce its investments in public infrastructure. For example, while Mankiw and Weinzierl find that 50% of a tax cut on capital income is self-financing, Ferede concludes that only 6% of the tax cut would be self-financing if the tax cut meant the government had to cut back on productive spending. In a quantitative exercise below, I show that around 20% of a tax cut is self-financing in a Romer-style model.

The AK model as described above relies on large spillovers from using capital to generate endogenous growth through capital accumulation as well as being consistent with facts about the share of income paid to capital. Furthermore, empirical evidence on the effect of the size of government on the economy's growth rate does not come down strongly in favor of such strong scale effects (Easterly and Rebelo (1993); Mendoza et al. (1997); Jones (1995b)). While this may be consistent with appropriately parametrized AK models, as in Stokey and Rebelo (1995), it is also consistent with the model I present in which tax rates do not affect the steady-state growth rate of the economy, but may affect the steady-state level of activity.

The paper proceeds as follows. Section 2 presents the Romer-style model with taxes on capital income and discusses the steady-state and transition dynamics in this model. Section 3 presents comparative dynamic responses of tax revenue to tax rates. Section 4 concludes.

2. The Romer Model with Capital Income Taxes

2.1. The Economic Environment and Agents

The economic environment consists of three production sectors. Final goods are produced using durable intermediate goods and labor. The intermediate goods are produced using final output (in the form of capital) and designs.

These designs come from the research and development sector, which uses labor and previously developed designs in production, though existing designs used in the R&D sector are not compensated in the decentralized allocation considered here.

The production of new designs used for making intermediate goods proceeds according to

$$\dot{A}_t = \nu A_t^{\phi} L_{At}^{\lambda}, \quad \phi < 1, \lambda > 0, A_0 > 0, \nu > 0 \tag{1}$$

The intermediate goods sector uses capital together with designs to produce differentiated intermediate inputs. One unit of capital produces one unit of the intermediate good. Each intermediate goods producer owns the design used in production. The measure of designs is A_t. Total production of intermediate goods is determined by the size of the capital stock:

$$\int_0^{A_t} x_{it} di = K_t \tag{2}$$

Final output, which can be consumed or transformed into capital, is produced with intermediate inputs and labor

$$Y_t = \left(\int_0^{A_t} x_{it}^{\theta} di \right)^{\alpha/\theta} L_{Yt}^{1-\alpha} \tag{3}$$

The decentralized equilibrium in this economy features solutions to the following problems.

Household Problem. The household problem is to choose time paths of c_t (consumption) and v_t (financial assets) that maximize

$$\int_0^{\infty} e^{-(\rho-n)t} \frac{c_t^{1-1/\sigma} - 1}{1 - 1/\sigma} dt$$

taking the full time series of prices and taxes as given, and subject to the following constraints

$$\dot{v}_t = ((1-\tau_v)r_t - n)v_t + w_t - c_t + tr_t, \quad v_0 > 0 \tag{4}$$

$$\lim_{t\to\infty} v_t \exp\left\{-\int_0^t ((1-\tau_v)r_s - n)ds\right\} \geq 0 \quad NPG \tag{5}$$

where v is assets per person, c is consumption per person, w is the wage rate, r is the pre-tax return on assets, ρ discounts future utility, n is the growth rate of population, tr are net transfers from the government, and τ_v is the tax rate for asset income.[4] The assets, v are claims on both physical capital K and patented designs A.

Final Goods Problem. The final goods sector is perfectly competitive. At each point in time, firms demand labor and intermediate goods, taking wages and intermediate goods prices as given, to maximize

$$\left(\int_0^{A_t} x_{it}^{\theta} di\right)^{\alpha/\theta} L_{Yt}^{1-\alpha} - w_t L_{Yt} - \int_0^{A_t} p_{it} x_{it} di \tag{6}$$

Intermediate Goods Problem. Patent-holding firms in the intermediate goods sector choose a price p_{it} and quantity to produce $x(p_{it})$ to maximize profits

$$x(p_{it})(p_{it} - r_t - \delta) \tag{7}$$

Research and Development Problem. Firms in the R&D sector produce new designs that intermediate goods firms use to produce new intermediate inputs. There is free entry in this sector, but there are externalities. Firms perceive a constant returns to scale production function, ignoring diminishing returns to labor at the aggregate level in this sector. Increases in activity (L_A) generate something akin to congestion effects, lowering the marginal product of labor.

[4] I abstract from a menu of taxes that includes taxes on labor incomes and consumption. Without a labor-leisure choice, these taxes do not distort allocations. Altering capital income taxes may affect tax revenues gathered through the labor income and consumption taxes since different levels of capital income tax imply different degrees of capital accumulation and production. My analysis neglects these possible feedback effects.

Firms sell their patented designs for price P_{At}. They demand labor, paid at the economy-wide wage rate w_t, maximizing profits

$$P_{At}\bar{\nu}_t L_{At} - w_t L_{At} \tag{8}$$

where $\bar{\nu} = A^{\phi} L_A^{\lambda-1}$.

Government Budget. The government simply collects taxes and returns them to households as lump sum transfers:

$$tr_t = \tau_v r_t v_t \tag{9}$$

In this model there is neither government consumption nor public goods provision.[5] Households are Ricardian, so the timing of tax rebates is irrelevant to the households' decisions. Assuming the government rebates all revenues immediately means that we do not have to keep track of the government's asset position.

2.2. Definition of Equilibrium

The *decentralized equilibrium with taxes* in this Romer economy is a time path for quantities $\{c_t, L_{Yt}, L_{At}, L_t, A_t, K_t, Y_t, v_t, \{\pi_{it}\}_{i=0}^{A_t}, \{x_{it}\}_{i=0}^{A_t}, \bar{\nu}_t, tr_t\}_{t=0}^{\infty}$ and prices $\{P_{At}, \{p_{it}\}_{i=0}^{A_t}, w_t, r_t\}_{t=0}^{\infty}$ such that for all t:

1. c_t, v_t solve the household problem
2. $\{x_{it}\}_{i=0}^{A_t}$ and L_{Yt} solve the final goods firm problem
3. $\{p_{it}\}_{i=0}^{A_t}$ and $\{\pi_{it}\}_{i=0}^{A_t}$ solve the intermediate goods firm problem
4. L_{At} solves the research and development firm problem
5. $Y_t = \left(\int_0^{A_t} x_{it}^{\theta} di\right)^{\alpha/\theta} L_{Yt}^{1-\alpha}$

[5]See Barro (1990); Jones et al. (1993); Ferede (2008) and others for models where the government can provide productive public goods.

6. A_t follows from equation (1)

7. K_t satisfies $\int_0^{A_t} x_{it} di = K_t$

8. $\bar{\nu}_t$ satisfies the ideas production function: $\bar{\nu}_t = A_t^{\phi} L_{At}^{\lambda-1}$

9. Asset arbitrage: $r_t = \frac{\pi_{it}}{P_{At}} + \frac{\dot{P}_{At}}{P_{At}}$

10. r_t clears the financial market: $v_t L_t = K_t + P_{At} A_t$

11. w_t clears the labor market: $L_{Yt} + L_{At} = L_t$

12. $L_t = L_0 e^{nt}$

13. tr_t satisfies the government budget constraint: $tr_t = \tau_v r_t v_t$

Note that households are taxed on their income derived from assets. Financial income is derived from either physical capital or intellectual property (A). Asset arbitrage implies that the returns to investing a dollar in each asset class be the same. This is condition (9) in the definition of equilibrium above. In the presence of taxes, this condition implies that capital gains from appreciating prices of intellectual property are taxed. If only profits were taxed, the arbitrage equation would be:

$$(1 - \tau_v) r_t = (1 - \tau_v) \frac{\pi_{it}}{P_{At}} + \frac{\dot{P}_{At}}{P_{At}},$$

and this would have different implications for the steady-state price of patented ideas.

In fact, in the absence of depreciation, or if income from physical capital is taxed without allowing for depreciation, a policy that taxes only dividend payments and not capital gains of patented technologies makes the composition of the capital stock (i.e., the share of the overall capital stock that is physical capital distinct from intellectual property) invariant to the financial income tax rate.

2.3. Balanced Growth Path

This section presents some properties of the balanced growth path for the economy. Consider first static aspects of the equilibrium allocation. Each intermediate goods producer faces the same problem, so they will produce the same quantity x and sell it for the same price p. The profit π for each patent holder will be the same and all patents will trade at the same price P_A. Since the entire stock of physical is divided among the intermediate goods producers

$$x_{it} = x_t = \frac{K_t}{A_t}$$

and the price charged is a markup over marginal cost

$$p_{it} = p_t = \frac{1}{\theta}(r_t + \delta)$$

so that the profit for each firm is[6]

$$\pi_{it} = \pi_t = \frac{1-\theta}{\theta}(r_t + \delta)\frac{K_t}{A_t} = \alpha(1-\theta)\frac{Y_t}{A_t}.$$

Note that θ relates to the profit share. The share of final output (though not of total income) paid out as pure profits is $\alpha(1-\theta)$. If profits actually represent 10% of final output and α is one-third, then the appropriate value for θ would be about 0.7. The gross markup is $1/\theta$. So in order to match net markups of 10%, θ should be around 0.9. From this, I take 0.7 to 0.9 as a plausible range for values of θ.

As in Jones (1995a), the steady-state growth rate of A is given by[7]

$$g_A = \frac{\lambda n}{1-\phi}.$$

[6]Here we use the fact that $r_t + \delta = \alpha\theta Y_t/K_t$, where θ is the fraction of its marginal product that capital is paid.

[7]I use g_z to denote the balanced growth path growth rate of the variable z.

Since output is equal to

$$Y_t = A_t^{\alpha\frac{1-\theta}{\theta}} K_t^{\alpha} L_{Yt}^{1-\alpha}$$

the growth rate of output in steady state is given by

$$g_Y = g_K = n + \frac{\alpha}{1-\alpha}\frac{1-\theta}{\theta} g_A = \left(1 + \frac{\alpha}{1-\alpha}\frac{1-\theta}{\theta}\frac{\lambda}{1-\phi}\right) n$$

so that the growth rate of output and capital in steady state depends only on structural parameters, not on investment rates or tax rates.[8] In this model, there is no tradeoff between the current level of tax revenue and the steady-state growth rate of tax revenue as there is in the AK model.

From the capital accumulation equation

$$\dot{K}_t = Y_t - C_t \delta K_t$$

we know that consumption grows at the same rate as output and capital in steady state. Therefore, the consumption Euler equation determines the steady-state interest rate: from the household problem, the growth rate of consumption is

$$\dot{c}_t / c_t = \sigma((1-\tau_v) r_t - \rho) \tag{10}$$

$$\rightarrow g_Y - n \tag{11}$$

$$\Rightarrow r^* = \frac{\frac{\alpha}{1-\alpha}\frac{1-\theta}{\theta}\frac{g_A}{\sigma} + \rho}{1-\tau_v} \tag{12}$$

Capital income taxes do not affect the balanced growth rate of consumption. Higher tax rates raise the steady-state return to assets the household owns. Since the marginal product of capital is decreasing in the amount of capital, this means that the steady-state capital stock is lower. Similarly the stock of knowl-

[8]This is the distinguishing feature of semi-endogenous growth models. By contrast, first generation endogenous growth models (Romer (1990); Grossman and Helpman (1991); Aghion and Howitt (1992)) have strong scale effects so that the growth rate may be influenced by tax rates. See Jones (1999) and Jones (2005) for more on this point.

edge is also lower in a steady state with higher capital income taxes.

The fraction of labor allocated to the research and development sector is consistent with integrated labor markets. The wage paid to researchers is equal to the wage received by laborers producing final output. Therefore,

$$w_t = P_{At}\frac{\dot{A}_t}{L_{At}} = (1-\alpha)\frac{Y_t}{L_{Yt}} \tag{13}$$

which implies that

$$\frac{s_{At}}{1-s_{At}} = \frac{P_{At}\dot{A}_t}{(1-\alpha)Y_t}. \tag{14}$$

On the balanced growth path, asset arbitrage requires $P_{At} = \frac{\pi_t}{r^* - g_{P_A}}$, where $g_{P_A} = g_\pi = g_Y - g_A$. Consequently

$$\frac{s_A^*}{1-s_A^*} = \frac{\alpha(1-\theta)g_A}{(1-\alpha)(r^* - (g_Y - g_A))} \equiv \psi^*. \tag{15}$$

The steady-state share of labor allocated to research and development is

$$s_A^* = \frac{\psi^*}{1+\psi^*} = \frac{\alpha(1-\theta)g_A}{(1-\alpha)(r^* - (g_Y - g_A)) + \alpha(1-\theta)g_A}. \tag{16}$$

Of the terms in this equation, only the steady-state interest rate depends on the tax rate applied to capital income. Since higher interest rates lower the present value of future profits resulting from innovation, they reduce the price of a patented idea. Lower values of patents discourage the research and development required to develop new ideas, reducing s_A.[9]

The production function for new ideas shows that on the balanced growth path

$$A_t^* = \left[\frac{\nu L_t^\lambda s_A^{*\lambda}}{g_A}\right]^{\frac{1}{1-\phi}}. \tag{17}$$

Higher capital income taxes raise the interest rate and lower the fraction of

[9]Alternatively, higher tax rates cause less capital to be accumulated, raising its marginal product and therefore the interest rate. It follows that the share of labor working in R&D is lower the higher is the tax rate τ_v. As a result, the stock of knowledge is affected by τ_v.

workers producing new ideas. Therefore the balanced growth path stock of ideas is lower when tax rates are higher.

Along a balanced growth path, capital and output are determined from

$$\left(\frac{K}{Y}\right)^* = \frac{\alpha\theta}{r^* + \delta} \tag{18}$$

$$Y_t^* = A_t^{*\frac{\alpha}{1-\alpha}\frac{1-\theta}{\theta}} \left(\frac{K}{Y}\right)^{*\frac{\alpha}{1-\alpha}} (1 - s_A^*)L_t \tag{19}$$

$$K_t^* = \left(\frac{K}{Y}\right)^* Y_t \tag{20}$$

$$= A_t^{*\frac{\alpha}{1-\alpha}\frac{1-\theta}{\theta}} \left(\frac{\alpha\theta}{r^* + \delta}\right)^{*\frac{1}{1-\alpha}} (1 - s_A^*)L_t. \tag{21}$$

Increases in the financial income tax rate reduce A and K/Y but increase the fraction of workers producing physical output, so there are competing effects of capital income taxes on output. This mirrors the relationship between the optimal and equilibrium allocations in the Romer model. For some parametrizations the equilibrium involves overinvestment in R&D, while in others there is too little R&D (s_A is too small).[10] If their effect through labor market channels is strong enough, higher capital income taxes could actually increase the size of the stock of physical capital. and output See the calibration below for quantitative results in which a lower tax increases the capital stock and output, even though it diverts labor to the R&D sector.

The stock of assets includes both physical capital and patented ideas. The total value of these assets on the balanced growth path is

$$V_t^* = K_t^* + P_{At}^* A_t^* \tag{22}$$

$$= \left(\frac{\alpha\theta}{r^* + \delta} + \frac{\alpha(1-\theta)}{r^* - g_{P_A}}\right) Y_t^* \tag{23}$$

[10] For more, see Jones and Williams (1998, 2000) for a discussion of the social returns to R&D. Those papers discuss a related model that also includes a creative destruction distortion. Jones (2005) shows how the socially optimal rates of investment relate to the decentralized allocation's rates of investment in a model that does not have the creative destruction distortion.

so that the share of assets in the form of physical capital (versus patented ideas) depends on the steady state interest rate, which in turn depends on the tax rate on capital income.[11]

Since the value of innovations in the R&D sector are paid out to researchers as wages, changes in the allocation of labor and of the price of new ideas can affect the labor share of income. Note that total income in this model is $Y + P_A\dot{A}$. Payments to labor are $wL = (1-\alpha)Y + P_A\dot{A}$. A reduction in the tax rate on capital income lowers the real interest rate and raises the value of output in the R&D sector relative to the final goods sector. This in turn means the the labor share of income rises.

Tax revenue for the government is $\tau_v r^* V^*$. Of this, the tax base is $r^* V^*$. The Laffer conjecture in this context is that a reduction in the tax rate will cause the tax base to increase so much that the product of the two increases. Since a reduction in the tax rate causes the real interest rate to be lower in steady state, this would require a large increase in the value of assets.

[11]If $\delta = 0$ and $\alpha = \theta$, then $V_t^* = \alpha \frac{Y_t^*}{r^*}\left(\frac{r^* - \alpha n}{r^* - n}\right)$. In that case the asset structure of the economy depends on the growth rate of population and on the steady-state interest rate, which may respond to capital income taxes. Assuming there is no population growth, the share of assets that are physical capital is α, independent of τ_v. More generally, the effect of the population growth rate on the composition of assets depends on other parameters in the model. If $\alpha = \theta$, then higher n causes the growth rate of the price of an idea to be higher. This lowers the current price of a new idea and means more of the stock of assets will be physical capital. If $\alpha < \theta < 1$ and $\lambda > 1 - \phi$, entirely plausible values, it is possible for this effect to be reversed. For some such combinations of parameters higher population growth lowers the growth rate of the price of an idea, increasing its current price and the extent of investment in R&D.

2.4. Transition Dynamics

I log-linearize the key equations of the model as follows.[12] Define the vector γ as

$$\gamma_t = \begin{pmatrix} \gamma_{1t} \\ \gamma_{2t} \\ \gamma_{3t} \\ \gamma_{4t} \end{pmatrix} \equiv \begin{pmatrix} \log(C_t/K_t) \\ \log(\bar{Y}_t/K_t) \\ \log(s_{At}) \\ \log(\bar{\dot{A}}_t/A_t) \end{pmatrix} \tag{24}$$

where $\bar{Y}$ is the maximum output that could be obtained at a point in time, based on setting s_A equal to zero, and $\bar{\dot{A}}$ is the maximum rate of change of A that is possible at a point in time, based on setting s_A equal to one.[13] I use maximum output instead of actual output so that this variable is a genuine state variable and is unable to jump. With this set-up there are two obvious state variables and two control variables that correspond to the two key allocation decisions in the model: to consume or invest, and to produce final output or to produce ideas. Furthermore, each component of γ_t is constant on the balanced growth path, so $\dot{\gamma}_t$ will converge to zero. Therefore,

$$Y_t = A_t^{\alpha\frac{1-\theta}{\theta}} K_t^{\alpha} L_t^{1-\alpha}(1 - s_{At})^{1-\alpha} = \bar{Y}_t(1 - s_{At})^{1-\alpha} \tag{25}$$

and

$$\bar{\dot{A}}_t = \nu A_t^{\phi} L_t^{\lambda} = \dot{A}_t s_{At}^{-\lambda} \tag{26}$$

The limiting values of these variables are determined as follows. Equation (16) determines the steady-state value γ_3^*. Then γ_4^* is equal to $\log(g_A(s_A^*)^{-\lambda})$. The

[12]More details are in appendix A.

[13]Arnold (2006) analyzes the dynamics of this model with $\lambda = 1$. He reduces the model to a similar set of variables as I do here. The main differences are that one of his variables corresponds roughly to P_A rather than s_A and the variable that represents the output-capital ratio in his paper uses actual output rather than maximum output. Schmidt (2003) also discusses transition dynamics in the Romer model.

steady-state interest rate in equation (12) determines the steady-state capital output ratio, which combined with s_A^* determines γ_2^*. Finally, $C/K = Y/K - \dot{K}/K - \delta$ which determines γ_1^*.

It is convenient to work with these four variables since they are each constant on a balanced growth path. Two correspond roughly to the state variables in the model (γ_4 relates to the stock of knowledge, γ_2 to the capital stock), and do not jump in response to shocks. By contrast, the other two correspond to control variables (γ_1 to the investment rate, and γ_3 to the intensity of research and development efforts) and can jump. The dynamics of the four variables are determined by two initial conditions (K_0 and A_0) and two endpoint conditions (the limiting behavior of C and s_A).

The rate of change of γ is given by

$$\dot{\gamma}_t = \begin{pmatrix} \frac{\dot{C}_t}{C_t} - \frac{\dot{K}_t}{K_t} \\ \\ \frac{\dot{Y}_t}{Y_t} - \frac{\dot{K}_t}{K_t} \\ \\ \frac{\dot{s_{At}}}{s_{At}} \\ \\ \lambda \frac{\dot{L}_t}{L_t} - (1-\phi)\frac{\dot{A}_t}{A_t} \end{pmatrix} \tag{27}$$

Equations for three elements of this vector are straightforward. The derivation of each equation is covered in the appendix. The growth rate of consumption is given by the household's Euler equation. The growth rate of the capital stock comes from the capital accumulation equation. The growth rate of the maximum growth rate of A is determined by the growth rate of A and of population.

The growth rate of s_{At} is more complicated, and is based on the dynamics of the labor market equilibrium condition in equation (14). This equation implies that the rate of change of s_A is influenced by the rate of change of P_A, A, K, and L. For example, if the price of patented ideas is rising over time then, all else equal, the fraction of labor allocated to R&D will also be rising.

For all the calibrations I applied, the log-linearized system of equations was characterized by two negative and two positive eigenvalues so is saddlepath stable. This is consistent with there being two state variables and two jump variables (C and s_A). Arnold (2006) shows that a slightly simpler version of this model without taxes *must* have two negative and two positive eigenvalues.

3. Comparative Dynamics: Response to τ_v Changes

This section discusses the response of the economy in general and tax revenues in particular when there is a change in the financial income tax rate. It shows the long-run response of tax revenues to tax rates as well as transition paths for a range of variables.

3.1. Short-run Response of Tax Revenue

In the Ramsey model, the interest rate at a point in time is determined by the capital stock. Factor supplies are inelastic in the short-run, so the marginal product of capital is a given. Therefore the elasticity of tax revenue with respect to the capital income tax rate is equal to one in the Ramsey model in the short run (Mankiw and Weinzierl (2006)).

In the Romer model, the marginal product of capital depends on the allocation of labor between the two sectors. And even if the real interest rate were not to jump in the R&D model, if the price of a patented idea jumps, then the stock of assets whose income streams are taxed also jumps so that the elasticity of tax revenue with respect to changes in the tax rate need not be one. For some parametrizations, tax revenue jumps less than the percentage of the change in the tax rate, while in other parametrizations it jumps more.

3.2. Long-run Response of Tax Revenue

The long-run response of tax revenue to the tax rate on financial income depends mainly on a small number of key parameters. First, θ, which governs the substitutability in production of different kinds of capital goods, has a particularly important role. For low values of θ, low tax rates are consistent with high tax revenues, so there is a relevant Laffer curve effect. Evidence on the share of income received as pure profits (returns to patents) and on markups suggest that such values of θ are implausible. For higher values of θ (closer to 0.9 so that markups are around 10%) suggest that tax revenues are maximized at tax rates closer to 85%.[14]

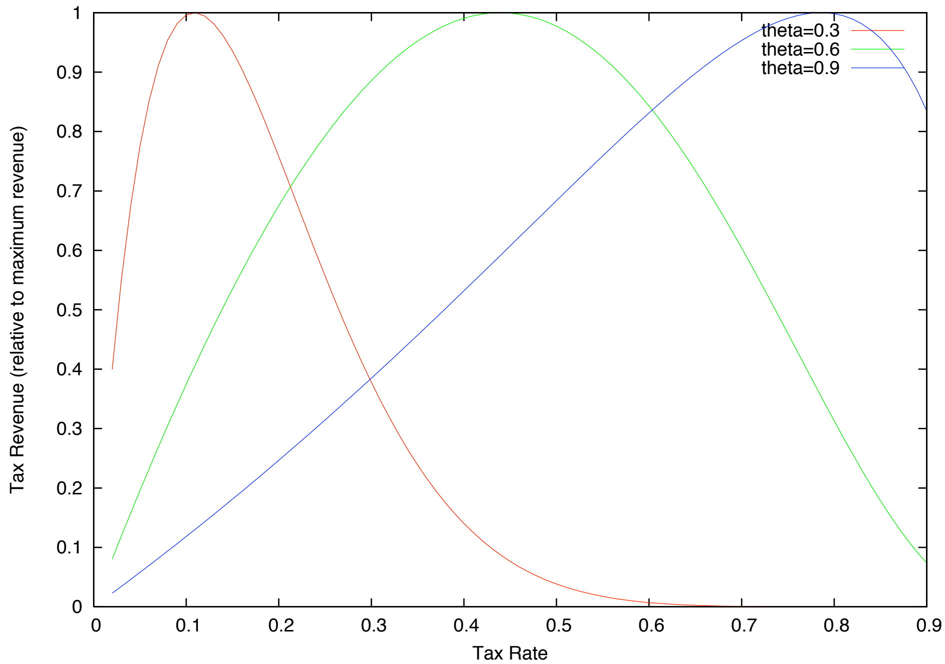
Figure 1 shows steady-state tax revenue as a function of the tax rate for three different values of θ. For high values of θ the long-run elasticity of output with respect to A is low. As a consequence, lower tax rates do not raise government revenue in general, even though they increase the stock of knowledge and hence output. For lower values of θ, tax revenue peaks as a function of the tax rate at relatively moderate tax rates. Figure 2 shows the tax rates that maximize tax revenue as a function of θ.

Low values of θ imply that a large share of income is accrued as pure profits. In the model, profits are $\alpha(1-\theta)$ fraction of final output. If α is $1/3$ and θ is 0.3, profits will be nearly twenty five percent of final output. In addition, the gross markup charged by producers of the differentiated intermediate goods is $1/\theta$, so low values of θ imply high markups. Evidence from Basu and Fernald (1997) and Broda and Weinstein (2006) suggests that markups in practice are much lower, perhaps around 10% to 20%, and profit shares are small.

Baseline parameter values used to generate these figures are reported in Table 1. The parameters imply that around 10% of final output is paid out as pure profits ($\alpha(1-\theta) = \frac{1}{3}(1-0.7) = 0.1$. Of the other parameters, most are standard. The exceptions are λ and ϕ. There is little empirical research to guide the

[14] By contrast, in the Ramsey model, tax revenues are maximized when $\tau_v = 1-\alpha$ where α is the elasticity of ouptut with respect to capital.

Figure 1: Steady-State Tax Revenue as a Function of the Tax Rate

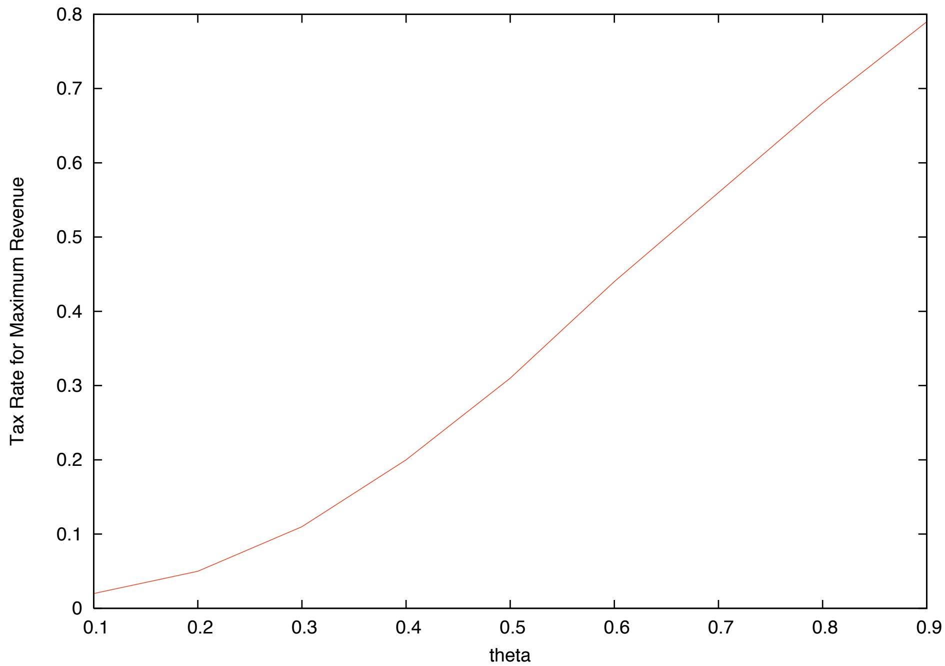


Notes: This graph plots steady-state tax revenue as a fraction of the maximum possible (over various tax rates) steady-state tax revenue for the relevant set of parameters.

calibration of these parameters. In the model, the steady-state growth rate of output per person is $\frac{\alpha}{1-\alpha}\frac{1-\theta}{\theta}\frac{\lambda}{1-\phi}n$. Using the other parameter values from table 1, and assuming a steady-state growth rate of output per worker of 2%, this suggests $\lambda \approx 9(1-\phi)$. A problem with this approach to calibrating λ and ϕ is that the historical growth rate of output may not be close to the steady-state growth rate (Jones (2002)).

The subsequent figures, 3(a) and 3(b), show steady-state output and consumption as functions of the tax rate, again for different values of θ. Since taxing financial income does not correct the underlying distortions in this economy, higher taxes are associated with lower output and consumption in the model, at least for these calibrations. That is, the optimal tax rate is negative. A key parameter here is ϕ, which is set to 0.9 indicating large dynamic spillovers from past research and development on present R&D productivity. Because of this

Figure 2: Revenue Maximizing Tax Rates as a Function of θ

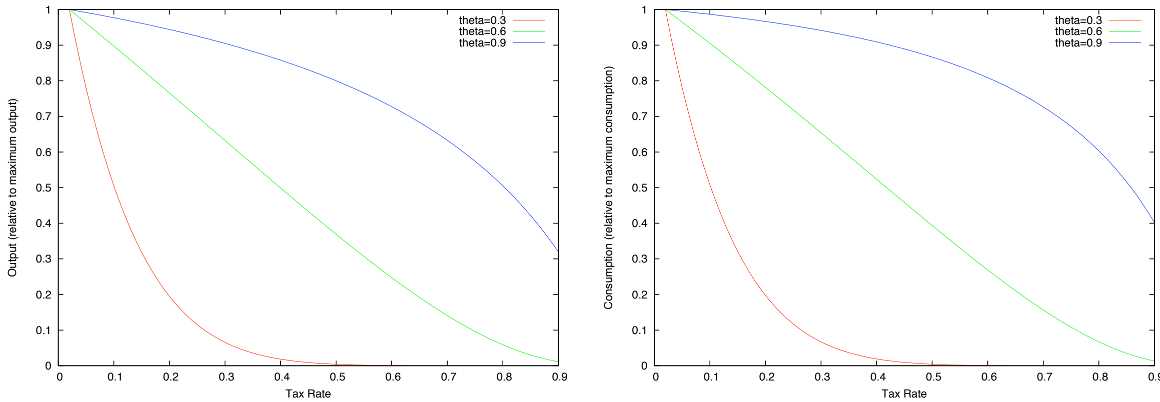


Notes: for other parameters at values indicated in table 1, this graph shows how the tax rate that maximizes tax revenue is related to the parameter θ.

Table 1: Calibrated Values of Model Parameters

Parameter	Meaning	Calibrated Value
α	Elasticity of Y w.r.t. K	1/3
θ	Related to Elasticity of Substitution between Varieties of Capital	0.7
δ	Depreciation rate for physical capital	0.05
ν	Productivity in R&D	1
λ	Elasticity of $\dot{A}$ w.r.t. L_A	0.7
ϕ	Elasticity of $\dot{A}$ w.r.t. A	0.9
ρ	Discount rate	0.02
σ	Intertemporal elasticity of substitution	1
n	Population growth rate	0.01

Figure 3: Steady-State Output and Consumption



(a) Steady-State Output as a Function of the Tax Rate

(b) Steady-State Consumption as a Function of the Tax Rate

distortion, as well as the usual monopoly distortion in the intermediate goods sector, the optimal policy in this model typically requires a subsidy to R&D effort.

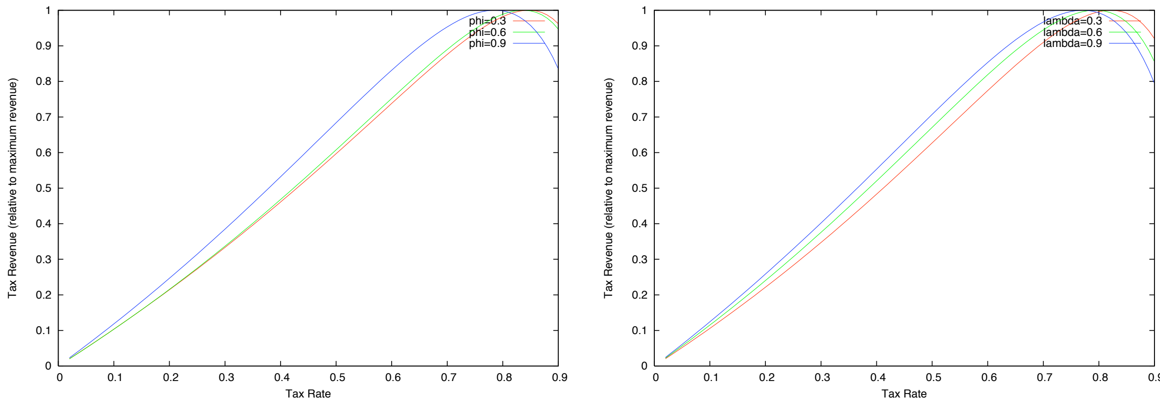
Figures 4(a) and 4(b) show how steady-state tax revenue responds to the tax rate for different values of ϕ and λ. Most significant about these graphs is that variation in ϕ is less important than variation in the tax rate, since the lines in figure 4(a) are close together.

Figure 5 shows that as θ increases, tax cuts are less likely to generate large revenue gains, or even large offsetting tax base expansions. As λ increases, revenue gains are more likely, at least for lower values of θ. While there is some evidence that is relevant to the appropriate value of θ, it is less clear how to calibrate λ in conjunction with ϕ. Are the stepping on toes effects important or not? What is a reasonable guess at the long-run growth rate of the economy, since the ratio $\lambda/(1-\phi)$ is important in determining this growth rate.

3.3. Dynamic Response to a Cut in Tax Rate

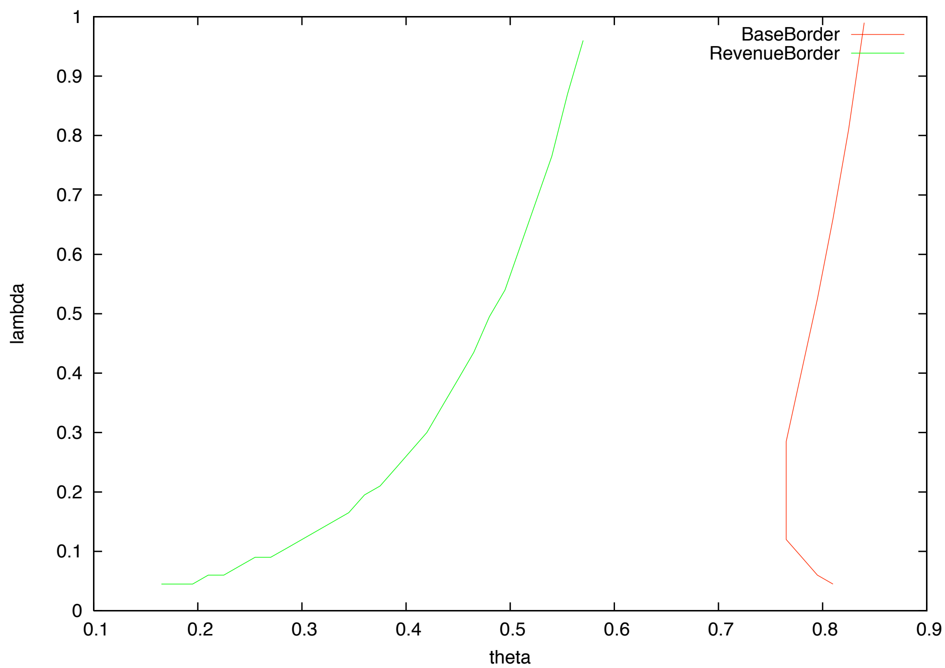
This section illustrates the response of the economy to a reduction in financial income tax rates from 40% to 30%. As in the set-up above, there are no other

Figure 4: Steady-State Tax Revenue as a Function of the Tax Rate



(a) For values of ϕ (b) For values of λ

Figure 5: Thresholds for Long-run Revenue Neutrality and Tax Base Neutrality of a Tax Cut



Notes: the Base Border curve indicates combinations of θ and λ that are consistent with a tax cut inducing no change in the size of the tax base in the long run. The Revenue Border curve indications combinations of the parameters that are consistent with not long-run change in tax revenue in response to a tax cut. (That is, the economy is at the peak of the Laffer Curve.) These curves are constructed based on a change in the tax rate from 40% to 30%.

taxes. The economy starts on its balanced growth path, then faces a new, permanently lower tax rate. The dynamic response of the economy is computed using the log-linearized version of the model.

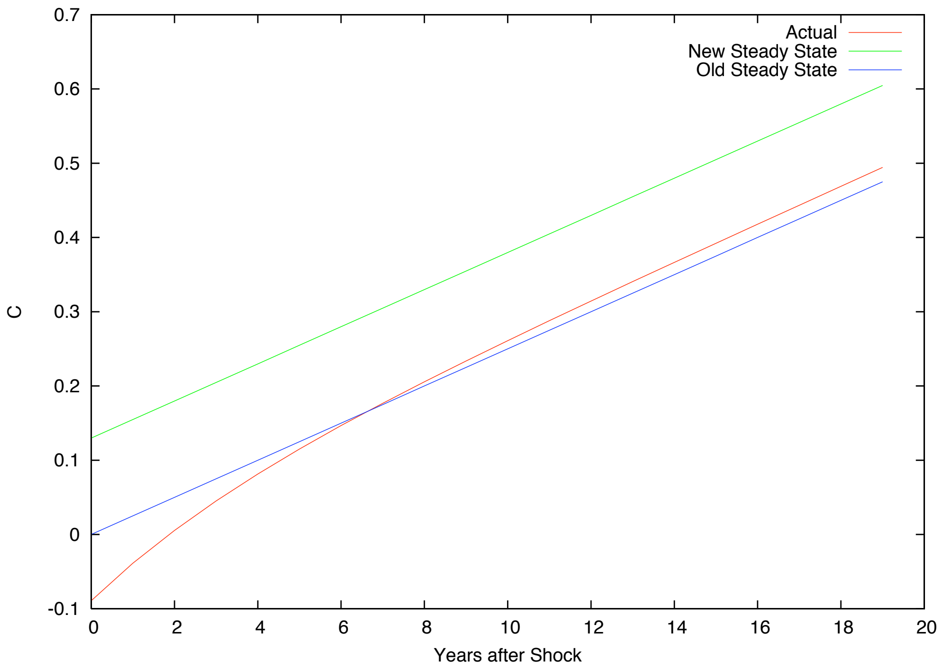
Figures 6 and 7 show the responses of the two key allocation choices in the economy, consumption and the allocation of labor between the two sectors. Initially consumption drops around 10%, but quickly rises to be above the previous balanced growth path, eventually converging to the new steady-state with consumption around 13% higher than it would have been without the tax reform. This cut in current consumption is a response to the suddenly higher after-tax returns available. Consumers are willing to reduce current consumption because, more than at the previous, higher tax rate, saving increases their assets and future incomes.

The response of the labor share working in R&D is more subtle. Initially the share working in R&D jumps up toward the new steady state. But after the jump the share gradually falls before eventually converging to the new steady-state value. This non-monotonic convergence is due to the dynamics of the system being governed by two negative eigenvalues. For s_A the signs of the coefficients on the corresponding eigenvectors are opposite, hence the initial drift away from the steady state before convergence. Appendix B gives more details.

Figure 8 shows the price of patented inventions jumping up from its initial balanced growth path, though it eventually converges to a level below its prior trajectory.

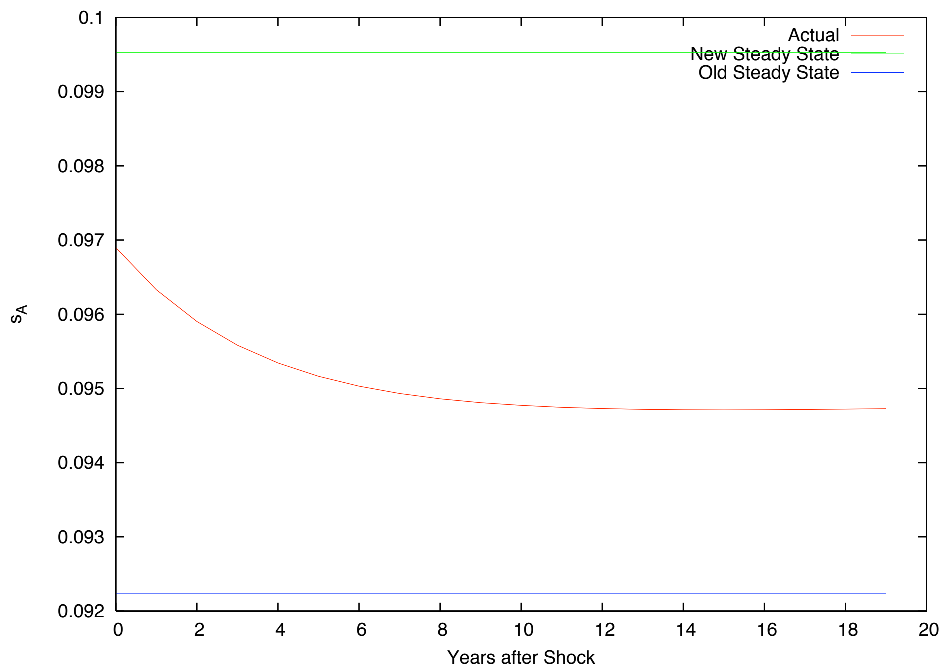
The tax revenue generated for the government falls initially by over 25%. The reduction is mainly because the tax rate is reduced, but also partly because of the reallocation of labor toward the R&D sector. Tax revenue continues to grow more slowly than its steady-state growth rate for some time, falling further below the new balanced growth path to which it eventually converges. Figure 10 shows the dynamic response of the tax base. The tax base initially falls relative to its counterfactual path in the old steady state. Eventually it expands to by around 8% higher than it would have been otherwise. The 8% expansion in the

Figure 6: Consumption Response to a Lower Tax Rate



Notes: This figure plots the log of actual consumption, steady-state consumption, and the previous steady-state consumption relative to the prior steady-state consumption in the initial period. One hundred times any observation gives the approximate percentage difference of that series from the starting point.

Figure 7: Labor Allocation Response to a Lower Tax Rate



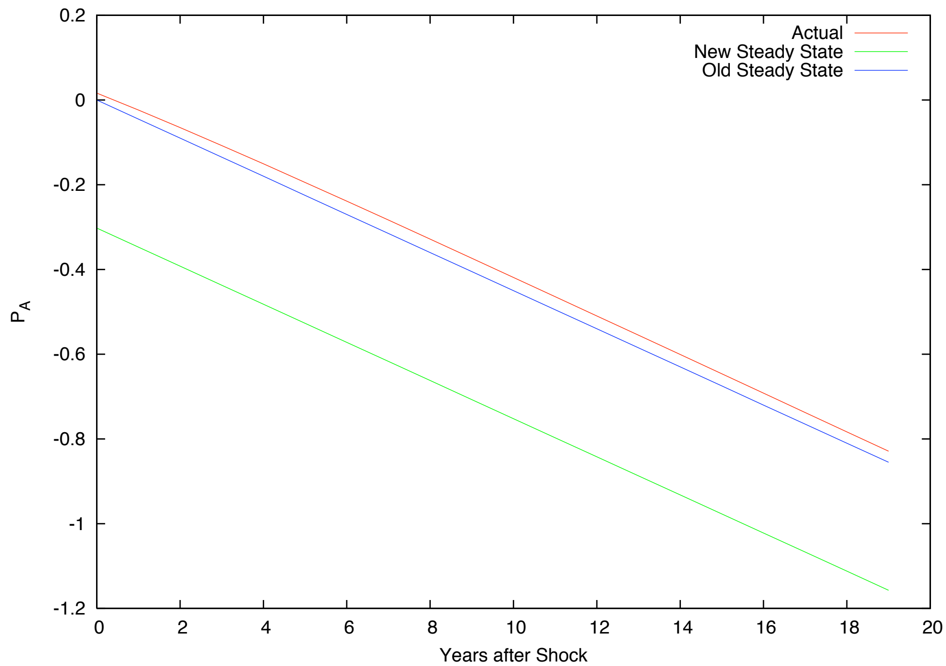
tax base makes up about 20% of the revenue lost from the reduction in the tax rate.

The convergence of the tax base and revenue, of P_A, and of the stock of designs is slow. The gradual convergence is influenced strongly by the value of ϕ. The high value of ϕ means that current investments in R&D stimulate future productivity in the R&D sector, so that diminishing returns do not set in very quickly in the accumulation of A by comparison with the accumulation of K.

4. Conclusion

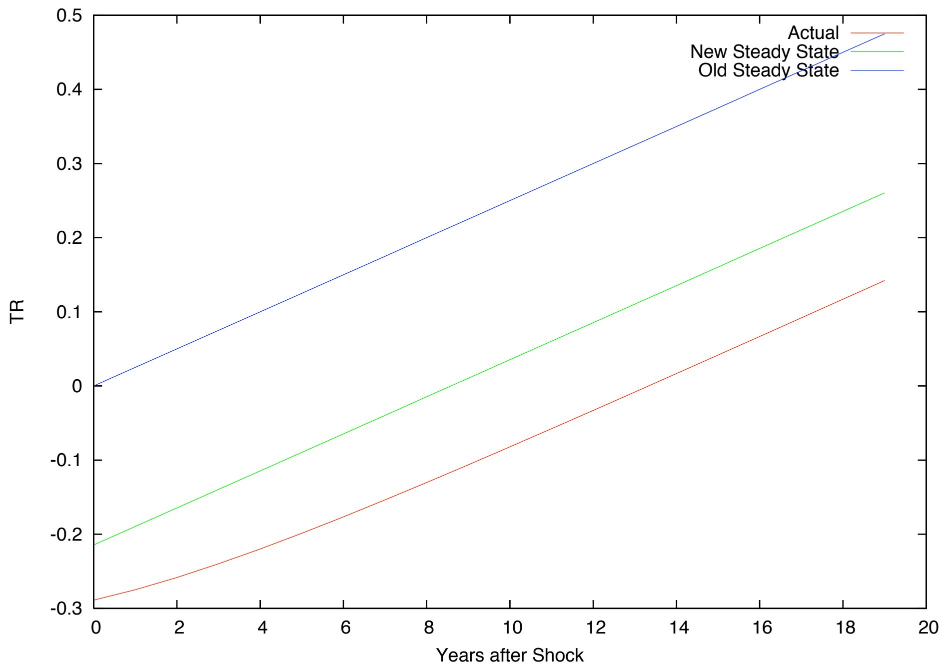
This paper has investigated the dynamic response of tax revenue to changes in the tax rate applied to financial income in a model of endogenous growth due to research and development. The model modifies Romer (1990) and Jones (1995a) to incorporate a tax on capital income, without distinguishing between income derived from physical capital and pure profits that accrue to patent

Figure 8: Response of the Price of a Patent to a Lower Tax Rate



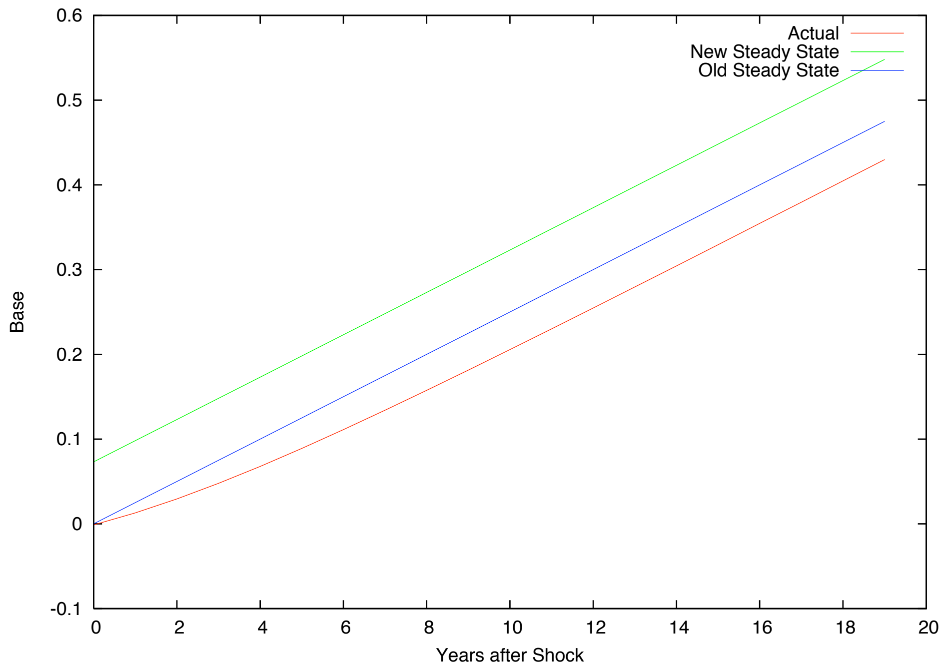
Notes: This figure plots the log of the market value of a new design P_A, steady-state P_A, and the previous steady-state P_A relative to the prior steady-state P_A in the initial period. One hundred times any observation gives the approximate percentage difference of that series from the starting point.

Figure 9: Response of Tax Revenue to a Lower Tax Rate



Notes: This figure plots the log of actual tax revenue, steady-state tax revenue, and the previous steady-state tax revenue relative to the prior steady-state tax revenue in the initial period. One hundred times any observation gives the approximate percentage difference of that series from the starting point.

Figure 10: Response of Tax Base to a Lower Tax Rate



Notes: This figure plots the log of the tax base, steady-state tax base, and the previous steady-state tax base relative to the prior steady-state tax base in the initial period. One hundred times any observation gives the approximate percentage difference of that series from the starting point.

holders. The log-linearized model is used to estimate the dynamic response of the economy to a tax cut.

Low values of θ generate significant Laffer curve effects, where a reduction in the tax rate stimulates the economy so much that tax revenue increases. However, such low values of θ are inconsistent with evidence on profit shares and markups. Instead, for more plausible parameter values, about 20% of a tax rate reduction is self-financing. The convergence to this new balanced growth path with a higher tax base can be very gradual, with the speed of convergence slowed down by the strong spillovers of current research output to future research productivity. These results suggest that widespread increases in tax rates across the developed world need not lead to reductions or only small increases in government revenue that will likely be needed to finance social services for retiring baby boomers.

A Log-Linearizing the Model

A1. Rate of Change

From the household's Euler Equation, we know that

$$\frac{\dot{C}_t}{C_t} = \sigma((1-\tau_v)r_t - \rho) + n \tag{28}$$

where

$$\begin{aligned} r_t &= \alpha\theta\frac{Y_t}{K_t} - \delta &(29)\\ &= \alpha\theta\frac{\bar{Y}_t}{K_t}(1-s_{At})^{1-\alpha} - \delta &(30)\\ &= \alpha\theta e^{\gamma_{2t}}(1-e^{\gamma_{3t}})^{1-\alpha} - \delta &(31)\end{aligned}$$

The capital accumulation equation is standard and gives

$$\begin{aligned} \frac{\dot{K}_t}{K_t} &= \frac{Y_t}{K_t} - \frac{C_t}{K_t} - \delta &(32)\\ &= e^{\gamma_{2t}}(1-e^{\gamma_{3t}})^{1-\alpha} - e^{\gamma_{1t}} - \delta. &(33)\end{aligned}$$

Therefore,

$$\dot{\gamma_{1t}} = \sigma((1-\tau_v)(\alpha\theta e^{\gamma_{2t}}(1-e^{\gamma_{3t}})^{1-\alpha} - \delta) - \rho) + n - e^{\gamma_{2t}}(1-e^{\gamma_{3t}})^{1-\alpha} + e^{\gamma_{1t}} + \delta \tag{34}$$

The second element of γ changes according to the growth rates of $\bar{Y}$ and K. Note that maximum output can be written as

$$\bar{Y}_t = A_t^{\frac{\alpha}{1-\alpha}\frac{1-\theta}{\theta}}\left(\frac{K_t}{\bar{Y}_t}\right)^{\frac{\alpha}{1-\alpha}} L_t \tag{35}$$

so the growth rate of $\bar{Y}$ is

$$\frac{\alpha}{1-\alpha}\frac{1-\theta}{\theta}\frac{\dot{A}_t}{A_t}+\frac{\alpha}{1-\alpha}\left(\frac{\dot{K}_t}{K_t}-\frac{\dot{\bar{Y}}_t}{\bar{Y}_t}\right)+n \tag{36}$$

This implies that

$$\begin{aligned}
\dot{\gamma_{2t}} &= \frac{\alpha}{1-\alpha}\frac{1-\theta}{\theta}\frac{\dot{A}_t}{A_t}-\frac{\alpha}{1-\alpha}\left(\frac{\dot{K}_t}{K_t}-\frac{\dot{\bar{Y}}_t}{\bar{Y}_t}\right)+n-\frac{Y_t}{K_t}+\frac{C_t}{K_t}+\delta && (37)\\
&= \frac{\alpha}{1-\alpha}\frac{1-\theta}{\theta}e^{\lambda\gamma_{3t}+\gamma_{4t}}-\frac{\alpha}{1-\alpha}\dot{\gamma_{2t}}+n-e^{\gamma_{2t}}(1-e^{\gamma_{3t}})^{1-\alpha}+e^{\gamma_{1t}}+\delta && (38)\\
&= \alpha\frac{1-\theta}{\theta}e^{\lambda\gamma_{3t}+\gamma_{4t}}+(1-\alpha)(n-e^{\gamma_{2t}}(1-e^{\gamma_{3t}})^{1-\alpha}+e^{\gamma_{1t}}+\delta) && (39)
\end{aligned}$$

The rate of change of γ_4 is straightforward also.

$$\begin{aligned}
\dot{\gamma_{4t}} &= (\phi-1)\frac{\dot{A}_t}{A_t}+\lambda\frac{\dot{L}_t}{L_t} && (40)\\
&= -(1-\phi)e^{\lambda\gamma_{3t}+\gamma_{4t}}+\lambda n && (41)
\end{aligned}$$

The rate of change of γ_{3t} is equal to

$$\begin{aligned}
\dot{\gamma_{3t}} &= \frac{1}{1-\lambda+\alpha\frac{e^{\gamma_{3t}}}{1-e^{\gamma_{3t}}}}((1-\tau_v)(\alpha\theta e^{\gamma_{2t}}(1-e^{\gamma_{3t}})^{1-\alpha}-\delta)-(1-\alpha-\lambda)n && (42)\\
&\quad -\alpha(e^{\gamma_{2t}}(1-e^{\gamma_{3t}})^{1-\alpha}-e^{\gamma_{1t}}-\delta) && (43)\\
&\quad +e^{\lambda\gamma_{3t}+\gamma_{4t}}(\phi-\alpha\frac{1-\theta}{\theta}+(1-\theta)\frac{\alpha}{1-\alpha}\frac{1-e^{\gamma_{3t}}}{e^{\gamma_{3t}}})) && (44)
\end{aligned}$$

A2. Linearization

Linearize the transition equations above. Evaluate the Jacobian at the steady-state values.

$$\frac{\partial \dot{\gamma_{1t}}}{\partial \gamma_{1t}} = e^{\gamma_{1t}} \tag{45}$$

$$\frac{\partial \dot{\gamma_{1t}}}{\partial \gamma_{2t}} = e^{\gamma_{2t}}(1-e^{\gamma_{3t}})^{1-\alpha}\left(\alpha\theta\sigma(1-\tau_v)-1\right) \tag{46}$$

$$\frac{\partial \dot{\gamma_{1t}}}{\partial \gamma_{3t}} = -(1-\alpha)e^{\gamma_{2t}}(1-e^{\gamma_{3t}})^{1-\alpha}\left(\alpha\theta\sigma(1-\tau_v)-1\right)\frac{e^{\gamma_{3t}}}{1-e^{\gamma_{3t}}} \tag{47}$$

$$\frac{\partial \dot{\gamma_{1t}}}{\partial \gamma_{4t}} = 0 \tag{48}$$

$$\frac{\partial \dot{\gamma_{2t}}}{\partial \gamma_{1t}} = (1-\alpha)e^{\gamma_{1t}} \tag{49}$$

$$\frac{\partial \dot{\gamma_{2t}}}{\partial \gamma_{2t}} = -(1-\alpha)e^{\gamma_{2t}}(1-e^{\gamma_{3t}})^{1-\alpha} \tag{50}$$

$$\frac{\partial \dot{\gamma_{2t}}}{\partial \gamma_{3t}} = \alpha\lambda\frac{1-\theta}{\theta}e^{\lambda\gamma_{3t}+\gamma_{4t}} + (1-\alpha)^2 e^{\gamma_{2t}}(1-e^{\gamma_{3t}})^{1-\alpha}\frac{e^{\gamma_{3t}}}{1-e^{\gamma_{3t}}} \tag{51}$$

$$\frac{\partial \dot{\gamma_{2t}}}{\partial \gamma_{4t}} = \alpha\frac{1-\theta}{\theta}e^{\lambda\gamma_{3t}+\gamma_{4t}} \tag{52}$$

$$\frac{\partial \dot{\gamma_{3t}}}{\partial \gamma_{1t}} = \frac{1}{1-\lambda+\alpha\frac{e^{\gamma_{3t}}}{1-e^{\gamma_{3t}}}}\alpha e^{\gamma_{1t}} \tag{53}$$

$$\frac{\partial \dot{\gamma_{3t}}}{\partial \gamma_{2t}} = \frac{1}{1-\lambda+\alpha\frac{e^{\gamma_{3t}}}{1-e^{\gamma_{3t}}}}\alpha e^{\gamma_{2t}}(1-e^{\gamma_{3t}})^{1-\alpha}((1-\tau_v)\theta-1) \tag{54}$$

$$\frac{\partial \dot{\gamma_{3t}}}{\partial \gamma_{3t}} = \frac{1}{1-\lambda+\alpha\frac{e^{\gamma_{3t}}}{1-e^{\gamma_{3t}}}}(\quad \alpha(1-\alpha)e^{\gamma_{2t}}(1-e^{\gamma_{3t}})^{1-\alpha}\frac{e^{\gamma_{3t}}}{1-e^{\gamma}_{3t}}(1-(1-\tau_v)\theta) \tag{55}$$

$$+e^{\lambda\gamma_{3t}+\gamma_{4t}}\lambda(\phi-\alpha\frac{1-\theta}{\theta}+(1-\theta)\frac{\alpha}{1-\alpha}\frac{1-e^{\gamma_{3t}}}{e^{\gamma_{3t}}}) - e^{\lambda\gamma_{3t}+\gamma_{4t}}\frac{\alpha}{1-\alpha}\frac{1-\theta}{e^{\gamma_{3t}}} \tag{56}$$

$$-\frac{\alpha}{1-e^{\gamma_{3t}}}\frac{e^{\gamma_{3t}}}{1-e^{\gamma_{3t}}}\frac{1}{1-\lambda+\alpha\frac{e^{\gamma_{3t}}}{1-e^{\gamma_{3t}}}}\dot{\gamma_{3t}} \tag{57}$$

$$\frac{\partial \dot{\gamma_{3t}}}{\partial \gamma_{4t}} = \frac{1}{1-\lambda+\alpha\frac{e^{\gamma_{3t}}}{1-e^{\gamma_{3t}}}}e^{\lambda\gamma_{3t}+\gamma_{4t}}\left(\phi-\alpha\frac{1-\theta}{\theta}+(1-\theta)\frac{\alpha}{1-\alpha}\frac{1-e^{\gamma_{3t}}}{e^{\gamma^{3t}}}\right) \tag{58}$$

$$\frac{\partial \dot{\gamma_{4t}}}{\partial \gamma_{1t}} = 0 \tag{59}$$
$$\frac{\partial \dot{\gamma_{4t}}}{\partial \gamma_{2t}} = 0 \tag{60}$$
$$\frac{\partial \dot{\gamma_{4t}}}{\partial \gamma_{3t}} = -(1-\phi)\lambda e^{\lambda\gamma_{3t}+\gamma_{4t}} \tag{61}$$
$$\frac{\partial \dot{\gamma_{4t}}}{\partial \gamma_{4t}} = -(1-\phi) e^{\lambda\gamma_{3t}+\gamma_{4t}} \tag{62}$$

We can write the linearized system as

$$(\gamma_t \dot{-} \gamma^*) \approx \Gamma(\gamma_t - \gamma^*) \tag{63}$$

$$\Gamma = \begin{pmatrix} \frac{r^*+\delta}{\alpha\theta} - g - \delta & \frac{r^*+\delta}{\alpha\theta}(\alpha\theta\sigma(1-\tau_v)-1) & -\frac{r^*+\delta}{\theta}(1-\theta)\frac{\lambda n}{1-\phi}\frac{\alpha\theta\sigma(1-\tau_v)-1}{\rho+\frac{g-n}{\sigma}-(g-g_A)} & 0 \\ (1-\alpha)(\frac{r^*+\delta}{\alpha\theta} - g - \delta) & -(1-\alpha)\frac{r^*+\delta}{\alpha\theta} & \frac{\lambda n}{1-\phi}\frac{1-\theta}{\theta}(\alpha\lambda+(1-\alpha)(r^*+\delta)) & \alpha\frac{1-\theta}{\theta}\frac{\lambda n}{1-\phi} \\ x\alpha(\frac{r^*+\delta}{\alpha\theta} - g - \delta) & x\alpha\frac{r^*+\delta}{\alpha\theta}((1-\tau_v)\theta-1) & \Gamma_{3,3} & \Gamma_{3,4} \\ 0 & 0 & -\lambda^2 n & -\lambda n \end{pmatrix} \tag{64}$$

where r^* is the steady-state interest rate from equation (12), and g is the steady-state growth rate of output, capital and consumption; $x = (1-\lambda+\frac{\alpha^2}{1-\alpha}(1-\theta)\frac{\lambda n}{1-\phi}\frac{1}{(1-\tau_v)r^*-(g-g_A)})^{-1}$, and

$$\begin{aligned}\Gamma_{3,3} &= x(\alpha(r^*+\delta)\frac{1-\theta}{\theta}\frac{\lambda n}{1-\phi}\frac{1-(1-\tau_v)\theta}{(1-\tau_v)r^*-(g-g_A)} + \frac{\lambda^2\phi n}{1-\phi} - \\ &\quad \frac{\alpha\lambda^2 n}{1-\phi}\frac{1-\theta}{\theta} + \lambda((1-\tau_v)r^*-(g-g_A)))\end{aligned}$$

and

$$\Gamma_{3,4} = x\left(\frac{\lambda^2\phi n}{1-\phi} - \frac{\alpha\lambda^2 n}{1-\phi}\frac{1-\theta}{\theta} + \lambda((1-\tau_v)r^*-(g-g_A))\right)$$

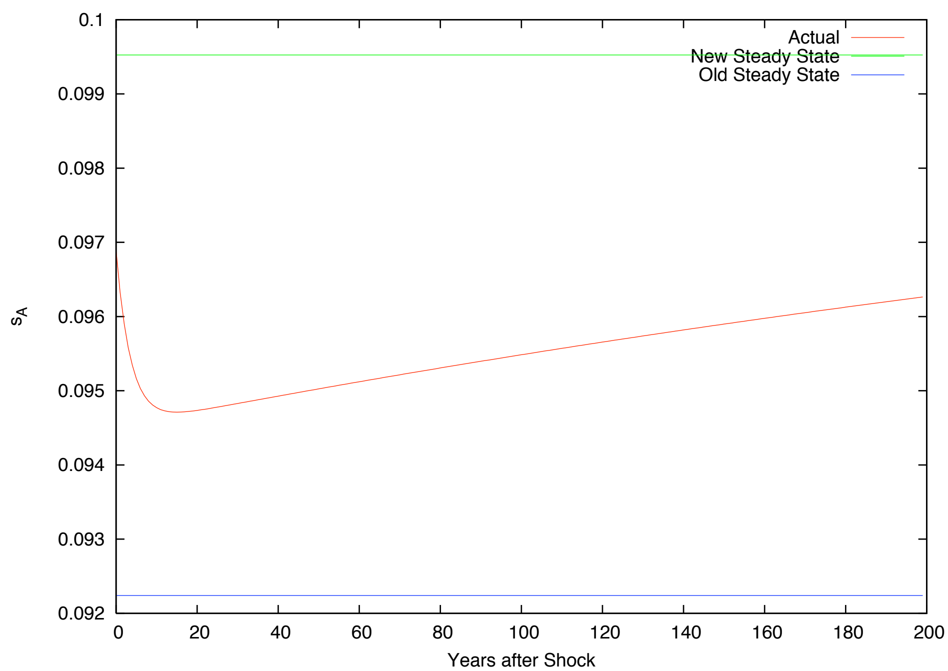
A3. Solutions of the Linearized System

The linearized system of equations is solved in Octave using the eigenvalue decomposition. Initial conditions for K and A generate the required boundary conditions to obtain the particular solution. In every case, two eigenvalues are negative (stable) and two positive. The non-monotonic dynamics of the system are due to the fact that there are two negative eigenvalues that determine the convergence behavior of the system, not one like in the Ramsey model.

B Longer Horizon Responses to a Tax Change

This graph confirms that s_A eventually converges to the new steady-state value. The share of labor working in the R&D sector converges slowly and non-monotonically. When the tax rate falls, s_A initially jumps up toward the new steady-state value. For several periods after that, as capital accumulates pushing up the marginal product of labor in final output, labor migrates back to the final output sector. With the passage of more time, the advance of the stock of designs increases (perceived) R&D productivity so that workers are drawn back toward the R&D sector.

Figure 11: Labor Allocation Response to a Lower Tax Rate



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