11-1-2010

Show me the Money: Intra-Household Allocation under Incomplete Information

Carolina Castilla
ccastilla, ccastilla@colgate.edu

Follow this and additional works at: http://commons.colgate.edu/econ_facschol

Part of the Economics Commons

Recommended Citation
http://commons.colgate.edu/econ_facschol/28

This Working Paper is brought to you for free and open access by the Economics at Digital Commons @ Colgate. It has been accepted for inclusion in Economics Faculty Working Papers by an authorized administrator of Digital Commons @ Colgate. For more information, please contact skeen@colgate.edu.
Show me the Money:
Intra-Household Allocation under Incomplete Information

Carolina Castilla\footnote{The author would like to thank Joyce Chen, Michael Sinkey and the participants at the 2010 AAEA Meetings for their helpful input and suggestions.}

November, 2010

Abstract:

There is evidence that individuals will sometimes withhold income transfers, such as bonuses, gifts, and cash transfers, from other members of the household (Ashraf (2009); Vogley and Pahl (1994)). In this paper, I show that the incentives to hide income under incomplete information over the quantity of resources available to the household differ for three different household resource management structures. I illustrate this with a simple two-stage game. In the first stage, one spouse receives a monetary transfer that is unobserved by her spouse, and she must decide whether to reveal or to hide it. In the second stage, spouses bargain over the allocation of resources between a household good and private expenditure. The three models differ in the resource allocation mechanism that takes place in second stage of the game: housekeeping allowance, independent management, and joint management. Results indicate that when one spouse receives a monetary transfer that is unobservable to her spouse, hiding is more likely to occur in households with a housekeeping allowance contract, compared to independent or joint management. In joint management households, however, a spouse may hide in equilibrium if the change in bargaining power associated with revealing the transfer is not significant enough to compensate for the loss in discretionary expenditure.

Key words: incomplete information, household bargaining, resource management systems.

JEL Classification: D13, D82, J12.
1. Introduction

Households are characterized by two main forms of interdependence between members: household public goods and caring, or how much one’s welfare is affected by the other’s. Household public goods can be thought of as those that benefit all members independently of who provides them, for instance investment in children’s human capital, such as education and health, provides welfare to both spouses even if it is the mother that makes sure her child gets the proper nutrition. It is often argued that, because families involve long-term, repeated interaction and caring, households will realize there are opportunities for Pareto improvement and therefore cooperation will evolve over time (Browning, et al., (2008)). However, these opportunities may diminish if information asymmetries exist and one spouse is able to exploit his or her information advantage without fear of detection.

Empirical studies on household bargaining, however, have found results consistent with non-cooperative behavior and inefficient allocation of resources within the household, which implies household public goods are being underprovided (Udry, (1996); Chen (2009); de Laat (2009); Ashraf, (2009)). Under-investment in household public goods, such as child human capital, has consequences on economic growth, welfare and can generate poverty traps. There is evidence that inadequate nutrition in childhood affects long-term physical development, as well as cognitive skills which in turn affect productivity later in life (Duflo, (2001)). Returns to education tend to be higher in developing countries and depend both on investment in educational attainment and in health due to the effects on productivity associated with proper nutrition. These human capital investments have important spillover effects in a household’s ability to step out of poverty because they increase child productivity providing alternative sources of income diversification to the household, in addition to the fertility effects through the trade-off of quality versus quantity of children, which also fosters economic development (Duflo, (2001); Rosenzweig, (1990)).
Cultural norms and informal institutions may generate inefficiencies, because they determine the household resource management structure and each spouse’s control over resources (Hoffman, 2009; Duflo and Udry, 2004; Anderson and Baland, 2002). For instance, Duflo and Udry (2004) find that, in Cote d’Ivoire, only the proceeds from yam production are allocated towards child human capital investments, whereas farm income from other men and women controlled crops is used for the owner’s private expenditures. Thus, if there is a negative shock affecting yam production, the decrease in the amount allocated towards child investments will not necessarily be compensated with resources coming from other sources. Anderson and Baland (2002) find that in poor households in Kenya men withhold a proportion of their income because it is commonly believed that they have the right to personal spending money. Hoffman (2009) finds that in Uganda, where Malaria is widespread, women when given cash to buy mosquito nets will use them on themselves because they perceive that by purchasing the nets they are buying usage rights, whereas when the nets are given to women for free they use them on their children.

There is a substantial sociological literature on the processes of intra-household decision making which emphasizes the importance of financial management structures in the family and the role that information can play in making decisions within a marriage (Wooley, 2001; Pahl (1994)). The information environment can further cause inefficiencies because cooperation may render unsustainable (Chen, 2009; De Laat, 2008; Udry, 1996). There is also evidence that individuals will sometimes withhold income transfers, such as bonuses, gifts, and cash transfers, from other members of the household. Vogler and Pahl (1994) find that husbands prefer bonuses to being paid for extra hours because they are able to maintain discretion on the way those additional resources are allocated. Ashraf (2009) finds that in the Philippines, when there is no fear of detection, husbands will withhold a monetary transfer from their wives and spend it on their private consumption. Income-hiding can have negative implications in the effectiveness of poverty
alleviation policies, as spouses wishing to successfully hide income must allocate resources away from household public goods, which are easily monitored, and opportunities to step out of inter-generational poverty will not be fully realized.

In what follows I show that the incentives to hide income under incomplete information over the quantity of resources available to the household differ for three different household resource management structures. I illustrate this with a simple two-stage game. In the first stage, one spouse receives a monetary transfer that is unobserved by her spouse, and she must decide whether to reveal or to hide it. In the second stage, spouses bargain over the allocation of resources between a household good and private expenditure. I develop three models that differ on the contract between spouses regarding the resource management system in the second stage, and show the conditions under which it is optimal for one spouse to hide a monetary transfer from the other.

The first model corresponds to the case where each spouse handles his or her own resources independently and makes individual contributions towards the household public good. The second model corresponds to a household under a housekeeping allowance system, where there is gender specialization. Both models have multiple equilibria: when both spouses make strictly positive contributions towards the household public good in the first model, or when the husband provides a strictly positive housekeeping allowance to his wife in the second, there exist incentives to hide income because the husband’s contribution is decreasing in his wife’s resources. There are also corner solutions that imply free-riding, where no incentives for hiding exist because one spouse’s allocations are unaffected by the other. The third model considers a collective household where spouses bargain over the way the joint pool of resources is allocated. An illustrative example assuming Cobb-Douglas utility is also provided.

The models’ results indicate that when one spouse receives a monetary transfer that is unobservable to her spouse, in equilibrium, income hiding can occur. The models further predict
that income hiding is more likely to occur in a household with a housekeeping allowance contract relative to both, an independent management and a joint management household. A joint management (or collective) household is the least likely to observe hiding, as long as the spouse without the information advantage has no dictatorial power. Further, in a collective household a spouse may hide in equilibrium if the change in bargaining power associated with revealing the transfer is not significant enough to compensate for the loss in discretionary expenditure.

The paper is organized as follows: in Section 2 a review of household resource control structures is presented; in Section 3 I describe the general characteristics of the models. In Sections 4 through 6 the models are developed, focusing on the conditions that must be met for income hiding to be an equilibrium outcome. In Section 7, I present some concluding remarks.

2. Household Resource Control Structures

The sociology literature has focused on developing a typology of household allocation systems (Pahl (1983)) that vary on whether spouses have separate or joint spheres of responsibility for managing household money, whereas economics focuses on modeling the decision-making process. Pahl identifies four allocation systems, three of which involve separate spheres, the whole wage system, the housekeeping allowance system and the independent management system, while the shared system involves joint spheres of responsibility (Pahl (1983)). These systems consist of an implicit or explicit contract of how the resources are managed within the household, and vary depending upon who has access or control over resources.

The whole-wage system can be divided into female and male managed. This system characterizes low income households, with labor specialization among spouses. In the male whole wage system, the husband manages all of the resources, such that the wife may not have any resources for
personal spending unless she has her own earnings. In actuality, this system is mostly observed in households where the wife is not employed. This case corresponds to the unitary model of the household, where there is a person that has all the bargaining power and thus makes all the decisions. In the female whole wage system, husbands hand all of their income minus the proportion they will use for personal spending to their wives and the wife is in charge of all the budgeting of the household thereafter (see Land (1969); Wilson (1987); Pahl (1983) in Pahl (1994)). In the housekeeping allowance system, the husband hands a fixed pre-contracted sum to his wife for household spending and he keeps the rest of his income for his own spending. This system is associated to middle class couples where the husband is the only earner (see Oakley (1974); Edwards (1981); Gray (1979); Pahl (1983) in Pahl (1994)). Both of these systems resemble the separate spheres model (Lundberg and Pollak (1993)), where there is gender specialization. However, these systems generate different incentives for income withholding. Gray (1979) found that, in households where the husband handed over his entire wage to his wife, he was less likely to earn money from overtime work relative to husbands who gave their wives a fixed housekeeping allowance. In the latter case extra earnings were retained by the husband, thus generating a greater incentive for him to do overtime (Gray, (1979) in Pahl, (1983)).

In the independent management system, each spouse handles her resources separately, and thus neither has access to all of the household money. This system resembles the voluntary contributions model, in the sense that each spouse decides on the optimal allocation of her own resources independently of her spouse. It has been found in households with high income levels where both spouses are earners and have similar levels of education (Browning, Chiappori and Lechene (2006); Chen and Wooley (2001)). Finally, in the pooling or joint management system both partners have access to all or nearly all of the household resources and both are thought to be responsible for management and expenditure from the common pool. This system is characteristic of middle income couples.
where both spouses work. This is the case of a collective household, where the partners bargain over the way the common pool is allocated.

Cultural and socio-economic characteristics determine the resource allocation management system within the household, which in turn establishes the distribution of responsibilities and control over resources. In what follows, I will argue that the incentives to exploit information asymmetries regarding income and expenditure depend on the implicit agreement regarding the resource allocation system within the household and the enforceability of these contracts.

3. General Description of the Family Decision Making Model

In the model there are two family members, \( f \) and \( m \). The household resource allocation decision is made in two stages. In the first stage household member \( f \) receives a monetary transfer \( T \) that is not perfectly observable to household member \( m \). The idea of the model is to mimic monetary transfers that are independent of household members’ labor market decisions, such as gifts, bonuses, or government transfers. Household member \( f \) has to decide whether to reveal that she received a transfer or to keep it for private consumption. For now \( T \) is assumed to be observable with probability zero and it is also assumed that \( m \) cannot observe \( f \)'s private consumption choices, though \( m \) can perfectly infer the increase in income through the public good allocation, which is perfectly observable. In the second stage, each household member makes his consumption choices conditional on the amount of the transfer member \( f \) revealed. The family decision-making process is solved by backwards induction. First, the consumption choices conditional on the amount of the transfer that becomes common knowledge are described, and then the circumstances under which it is optimal for \( f \) to hide the transfer are determined.
Both family members have preferences over consumption of one private (or personal) good, denoted \( x_i \), and one household public good, \( Q \). I assume that both family members face the same price for private goods which is normalized to 1 (one can think about the private good as being money for discretionary expenditure), and \( p \) is the price for the public good (\( Q \)). If both household members pool their incomes the joint budget constraint is:

\[
x_f + x_m + pQ = Y_f + Y_m + T
\]

(1)

If each member decides to allocate the income at her disposal separately, (\( Y_i \)) between private and household public goods, their individual budget constraint is:

\[
x_i + pQ_i = Y_i + T_i \quad \text{for } i = f, m
\]

(2)

Preferences over own consumption are represented by an egotistic utility function, \( U_i \). Utility depends on the aggregate level of consumption of household public goods (\( Q = Q_f + Q_m \)) and private goods (\( x \)). I assume that utility is separable in \( x \) and \( Q \):

\[
U_i = U(Q, x_i) = u(x_i) + v(Q) \quad \text{for } i = f, m
\]

(3)

The functions \( u(\cdot) \) and \( v(\cdot) \) satisfy the standard assumptions that \( u' > 0, v' > 0, u'' < 0, v'' < 0 \), and \( u'(0) = \infty, v'(0) = \infty \), implying \( x \) and \( Q \) are normal goods. In (3) I assume that the family members have the same functional form for simplicity, though in the examples provided I allow the different members to have different preferences over their private goods by specifying different preference parameters for each spouse. The characterization of goods as public or private depends on the nature of the good. The household public goods are assumed to be non-rival in utility, so they are of the Samuelson type. For instance, a clean house provides utility to both members of the household, while food provides utility only to the person that consumed it.

In what follows I develop three models that differ on the contract between spouses regarding the resource allocation mechanism, and show the conditions under which it is optimal for one spouse to hide a monetary transfer from the other.
4. Independent Resource Management Household

This section considers the case where spouses have an independent resource management system, which implies that no spouse has direct access to all household’s resources. I model this case using a voluntary contributions game. In this game each spouse decides separately how to allocate her resources between the household good and private consumption, taking the other’s contribution as given. I start by solving the benchmark case, before $f$ receives the monetary transfer ($T$). The optimization problem of spouse $i$ is to maximize the objective function (3) subject to her own budget constraint (2) (with $T=0$) and taking $j$’s household good purchases, $Q_j$, as given. Solving (2) for $x_i$ and substituting into (3) the optimization problem can be re-expressed as:

$$\max_{Q_i \geq 0} W_i = u(Y_i - pQ_i) + v(Q_i + Q_j) \quad \text{for } i,j = f,m$$

A non-negativity constraint is imposed on $Q_i$ because a corner solution is possible. The Kuhn-Tucker first-order conditions for this problem are:

$$\frac{aw_i}{aQ_i} = -p u'(Y_i - pQ_i) + v'(Q_i + Q_j) \leq 0$$

$$Q_i \frac{aw_i}{aQ_i} \leq 0; Q_i \geq 0$$

Because the problem is symmetric, solving the problem for each spouse yields the following reaction functions:

$$-p u'(Y_f - pQ_f) + v'(Q_f + Q_m) \leq 0 \quad \text{(5)}$$

$$-p u'(Y_m - pQ_m) + v'(Q_f + Q_m) \leq 0 \quad \text{(6)}$$

Solving the system of reaction functions that result from the Kuhn-Tucker conditions simultaneously yields the Nash equilibrium. Note that there are corner solutions, as well as interior solutions. Proposition 1 follows Wooley and Chen (2001) results to specify the conditions that must be met for an interior solution to exist.
Proposition 1: Given $Y_m$, there exists a $Y_f$ in the interval $(0,Y_m)$ such that the Nash equilibrium is a corner solution with $Q_f = 0$ if $Y_f \leq Y_f$; given $Y_f$, there exists a $Y_m$ in the interval $(0,Y_f)$ such that the Nash equilibrium is a corner solution with $Q_m = 0$ if $Y_m \leq Y_m$; and an interior solution exists with $Q_m, Q_f > 0$ when $Y_f > Y_f$ and $Y_m > Y_m$.

The properties of the corner solution equilibria are different from the properties of the interior equilibrium. In the corner solutions free-riding is observed, while in the interior solution both spouses make strictly positive contributions. These properties are demonstrated in Proposition 2, focusing on the implications for changes in $f$’s income.

4.1 Corner Equilibria:

Following Proposition 1, if $Y_f \leq Y_f \in (0,Y_m)$ or $Y_m \leq Y_m \in (0,Y_f)$ it is optimal for one of the household members to free ride. The conditions for free-riding depend on how one spouse’s income compares to his or her spouse’s, so unless their incomes are relatively close the public good will be underprovided, even within the context of the independent management contract. Proposition 2 states the properties of the equilibrium with respect to changes in $f$’s income and provides the foundations as to why in a free-riding equilibrium there are no incentives to hide a monetary transfer from $m$.

Proposition 2: In a free-riding equilibrium,

Case (i): If $Y_f \leq Y_f \in (0,Y_m)$ thus $Q_f = 0$, 

---

Browning, et al. (2006) show that for multiple public goods, at most both household members will contribute to one, and for the rest they will specialize in the provision based upon relative preferences towards the public goods. They show that when household members specialize, which is equivalent to the separate spheres case, these outcomes can be used as the non-cooperative threat points to the bargaining game. This is possible because unless both players make strictly positive contributions to all public goods, there is free-riding and so outcomes are inefficient, thus there can be gains from bargaining.
An increase in $Y_f$ results in $\frac{\partial x_f}{\partial Y_f} > 0; \frac{\partial q_f}{\partial Y_f} = \frac{\partial q_m}{\partial Y_f} = \frac{\partial x_m}{\partial Y_f} = 0$, while an increase in $Y_m$ results in

$\frac{\partial x_m}{\partial Y_m} > 0; \frac{\partial q_m}{\partial Y_m} > 0; \frac{\partial q_f}{\partial Y_m} = \frac{\partial x_f}{\partial Y_m} = 0$.

Case (ii): If $Y_m \leq Y_m^* \in (0, Y_f)$ thus $Q_m = 0$,

An increase in $Y_f$ results in $\frac{\partial x_f}{\partial Y_f} > 0; \frac{\partial q_f}{\partial Y_f} > 0; \frac{\partial q_m}{\partial Y_f} = \frac{\partial x_m}{\partial Y_f} = 0$, while an increase in $Y_m$ results in

$\frac{\partial x_m}{\partial Y_m} > 0; \frac{\partial q_m}{\partial Y_m} = 0; \frac{\partial q_f}{\partial Y_m} = \frac{\partial x_f}{\partial Y_m} = 0$.

If spouse $m$ is the sole provider of the public good, an increase in $Y_f$ will only impact $f$'s private consumption as long as $Y_f + \Delta Y_f \leq Y_f$. If spouse $f$ is the sole provider of the household public good, an increase in $Y_f$ doesn’t impact $m$’s private or public good consumption as long as $Y_m + \Delta Y_m \leq Y_m^*$. In either case, changes in $Y_f$ have no impact on $m$’s allocations. Now consider the case when $f$ receives a transfer ($T$) that is observable to household member $m$ with probability zero. Spouse $f$ then has to decide whether to allocate the monetary transfer ($T$) between private and household good consumption, thus directly or indirectly informing $m$ about the increase in her resources, or to hide it and spend it all on private consumption. If they are at a corner solution and the transfer does not increase $f$’s income enough to move to an interior solution, then there is no incentive to hide the transfer because a change in $Y_f$ only impacts $f$’s choices. However, the transfer can be such that $Y_f + \Delta Y_f > Y_f$ in which case the free-riding equilibrium for Case (i) would turn into an interior equilibrium.

\[3\] Two more cases are possible that are not examined in this paper. The first is when the transfer is such that an interior solution becomes possible when it is revealed. The second arises when the transfer increases the wife’s resources enough to move them from an interior to a corner solution.
4.2 Interior Equilibrium

The case in which both spouses make positive contributions to the household public good, from now on the interior equilibrium, is efficient within the context of the independent management contract. However, if public goods are provided via voluntary contributions, the solution is inefficient relative to what a social planner could achieve or even compared to a collective household. Nonetheless, within the context of an independent management agreement, the interior solution is the best they can do. This equilibrium is non-cooperative in the sense that no binding agreements are made, but it is self-enforcing as it is each spouse’s best response to make strictly positive contributions if the conditions \( Y_f > \bar{Y}_f \) and \( Y_m > \bar{Y}_m \) are met. This conditions imply that spouses income’s are similar, which aligns with evidence indicating that an independent management system is usually found among upper-middle class couples, with similar levels of education, where both spouses work (Vogler and Pahl, (1994)). Proposition 3 states the properties of the interior equilibrium.

**Proposition 3:** In an interior equilibrium, if \( Y_f > \bar{Y}_f \) and \( Y_m > \bar{Y}_m \), thus \( Q_m, Q_f > 0 \), an increase in \( Y_f \) results in \( \frac{\partial x_f}{\partial Y_f} > 0; \frac{\partial Q_f}{\partial Y_f} > 0; \frac{\partial Q_m}{\partial Y_f} < 0; \frac{\partial x_m}{\partial Y_f} > 0 \), while an increase in \( Y_m \) results in \( \frac{\partial x_f}{\partial Y_m} > 0; \frac{\partial Q_f}{\partial Y_m} < 0; \frac{\partial Q_m}{\partial Y_m} > 0; \frac{\partial x_m}{\partial Y_m} > 0 \).

When both spouses are making positive contributions (i.e. they are at an interior solution), an increase in \( Y_f \) increases both \( f \) and \( m \)'s private consumption, and \( f \)'s contribution to the public good, but \( m \)'s contribution decreases. This is the source of the incentives to hide income: as \( f \)'s resources increase, \( m \) reduces his contribution to the household good. Thus \( f \), can be made better off by hiding because it prevents \( m \) from reducing his contribution towards the public good, such that she can.
maintain the same household good consumption, in addition to the possibility of increasing her private consumption as well.

Now consider the case when \( f \) receives a transfer \((T)\) that is observable to household member \( m \) with probability zero. Spouse \( f \) then has to decide whether to reveal the transfer, and allocate it between private and household good consumption, or to hide it and spend it all on private consumption. If they are at an interior equilibrium, an increase in \( f \)'s resources decreases \( m \)'s contribution towards the public good and so if the conditions described in Proposition 4 are met, in equilibrium \( f \) will hide the transfer from \( m \).

**Proposition 4:** Given \( Y_f, Y_m \), when \( Y_f > \bar{Y}_f \) and \( Y_m > \bar{Y}_m \), there exists a threshold level of transfer \((\bar{T})\) such that for any \( T < \bar{T} \) the Subgame Perfect Nash Equilibrium of the game is to hide the transfer.

Proposition 4 stems from the comparison of the change in utility derived by \( f \) per unit of transfer, such that \( f \) will hide the transfer in equilibrium if the change in utility for every unit of \( T \) of revealing the transfer is less than the change in utility of not revealing the transfer and allocating it to private consumption. The decision to hide, however, depends on the size of the transfer: small transfers are likely to be hidden, whereas large transfers are likely to be revealed. The intuition for this result is that once a large enough transfer is received, on the margin, the reduction in the contribution of the spouse towards the public good becomes irrelevant. The following section provides a more intuitive result assuming preferences are of the Cobb-Douglas type.
4.3 Illustrative Example

An example can provide further intuition on the incentives to hide income. Consider the case of Cobb-Douglas preferences such that:

\[ U_f = U(Q, x_f) = \alpha_f \log(x_f) + (1 - \alpha_f) \log(Q) \]
\[ U_m = U(Q, x_m) = \alpha_m \log(x_m) + (1 - \alpha_m) \log(Q) \]

Where \( \alpha_f \) and \( \alpha_m \) are between 0 and 1. Each member then maximizes (7) subject to her own budget constraint (2), where \( T_m = 0 \) such that only spouse \( f \) receives a transfer. From the first-order conditions, the reaction functions take the following form:

\[ Q_i(Q_j) = \left[ \frac{(1-\alpha_i)Y_{i-\alpha_i}pQ_j}{p} \right] \quad \text{for } i = f, m = j \]  

Solving the system yields the Nash equilibrium private contributions to the household public good:

\[ Q^{VC^*}_f = \left[ \frac{(1-\alpha_f)(Y_f + T) - \alpha_f(1-\alpha_m)Y_m}{(1-\alpha_f)(1-\alpha_m)p} \right], \quad Q^{VC^*}_m = \left[ \frac{(1-\alpha_m)Y_m - \alpha_m(1-\alpha_f)(Y_f + T)}{(1-\alpha_f)(1-\alpha_m)p} \right] . \]  

(i) Corner Equilibria

Case (i): If \( Y_m \in \left[ 0, \frac{\alpha_m(1-\alpha_f)(Y_f + T)}{(1-\alpha_m)} \right] \) then \( Q^{VC^*}_m = 0 \), \( x^{VC^*}_m = Y_m \), \( x^{VC^*}_f = \alpha_f Y_f \), \( Q^{VC^*}_f = \frac{(1-\alpha_f)Y_f}{p} \).

Case (ii): If \( Y_f + T \in \left[ 0, \frac{\alpha_f(1-\alpha_m)Y_m}{(1-\alpha_f)} \right] \), i.e. if the transfer is such that an interior solution continues to be unfeasible, then \( Q^{VC^*}_f = 0 \), \( x^{VC^*}_f = Y_f \), \( x^{VC^*}_m = \alpha_m Y_m \), \( Q^{VC^*}_m = \frac{(1-\alpha_m)Y_m}{p} \).

As shown before, spouse \( m \)'s allocations do not respond to changes in spouse \( f \)'s resources in either case. Thus, if the transfer is such that \( (1 - \alpha_f)(Y_f + T) < \alpha_f (1 - \alpha_m)Y_m \) then \( m \)'s allocations do not change as \( f \)'s resources change and there are no incentives to hide the transfer.
(ii) Interior Equilibrium

Once spouse $f$ receives the transfer and if $Y_m \in \left[ \frac{\alpha_m(1-\alpha_f)Y_f}{(1-\alpha_m)}, \infty \right]$ and $Y_f + T \in \left[ \frac{\alpha_f(1-\alpha_m)Y_m}{(1-\alpha_f)}, \infty \right]$, then the optimal demands are given by:

\[
Q_f^{VC*} = \left[ \frac{(1-\alpha_f)(Y_f + T) - \alpha_f(1-\alpha_m)Y_m}{(1-\alpha_f a_m)p} \right], \quad x_f^{VC*} = \left[ \frac{\alpha_f(1-\alpha_m)(Y_f + T + Y_m)}{(1-\alpha_f a_m)} \right]
\]

\[
Q_m^{VC*} = \left[ \frac{(1-\alpha_m)Y_m - \alpha_m(1-\alpha_f)(Y_f + T)}{(1-\alpha_f a_m)p} \right], \quad x_m^{VC*} = \left[ \frac{\alpha_m(1-\alpha_f)(Y_f + T + Y_m)}{(1-\alpha_f a_m)} \right]
\]

If $f$ hides the transfer however, the household public good allocations do not change relative to the benchmark case (before the transfer occurs), and the entire transfer is allocated towards private expenditure. The demands are:

\[
Q_f^{VCH*} = \left[ \frac{(1-\alpha_f)(Y_f) - \alpha_f(1-\alpha_m)Y_m}{(1-\alpha_f a_m)p} \right], \quad x_f^{VCH*} = \left[ \frac{\alpha_f(1-\alpha_m)(Y_f + Y_m)}{(1-\alpha_f a_m)} \right] + T
\]

\[
Q_m^{VCH*} = \left[ \frac{(1-\alpha_m)Y_m - \alpha_m(1-\alpha_f)(Y_f)}{(1-\alpha_f a_m)p} \right], \quad x_m^{VCH*} = \left[ \frac{\alpha_m(1-\alpha_f)(Y_f + Y_m)}{(1-\alpha_f a_m)} \right]
\]

The decision to hide the transfer follows from the condition stated in Proposition 4, thus spouse $f$ hides $T$ iff:

\[
\left. \frac{\partial U_f}{\partial T} \right|_R = \left. \frac{\partial U}{\partial Q_f} \right|_R \frac{\partial Q_f}{\partial T} + \left. \frac{\partial U}{\partial x_f} \right|_R \frac{\partial x_f}{\partial T} = \frac{1}{Y_f + T + Y_m} < \frac{\alpha_f(1-\alpha_m)}{\alpha_f(1-\alpha_m)(Y_f + Y_m) + (1-\alpha_f a_m)T} = \left. \frac{\partial U_f}{\partial T} \right|_{NR}
\]

Simplifying yields the following condition,

\[
\frac{\alpha_f a_m}{(1-\alpha_f a_m)}(Y_f + Y_m) > T
\]
Thus, if there exists a transfer smaller than the proportion $\frac{\alpha_f \alpha_m}{\alpha_f \alpha_m + (1 - \alpha_f \alpha_m)} < 1$ of joint household income, spouse $f$ is better off hiding the transfer and allocating it to private consumption. Note that the transfer threshold level depends on relative preferences for private and public consumption between household members. In particular, the transfer threshold level increases as either $f$'s and/or $m$'s preference for private consumption increase relative to household good consumption. This is particularly interesting, because the decision to hide does not only depend on $f$'s relative preferences between private and public good consumption. If $m$ prefers private consumption more, he would be more likely to reduce his contribution towards the public good as $f$'s resources increase, also strengthening the incentives to hide.

5. **Housekeeping Allowance Resource Management Model**

In this section, the case where spouses adopt a housekeeping allowance system is considered, which corresponds to the case when there is gender specialization, such that the husband is in charge of providing money to the household, while the wife specializes in the provision of the public good. I model this case using a slightly simpler version of the Lundberg and Pollak (1993) separate spheres model, where the husband chooses the housekeeping allowance he gives his wife ($s$), and the wife chooses the household good allocation ($Q$). As in the independent management model, spouses do not commit to any binding agreements. The game has 3 stages: in the first stage, spouse $f$ receives a monetary transfer ($T$) that is unobservable to spouse $m$ and chooses whether to reveal the transfer or to hide it; in the second stage, spouse $m$ chooses the housekeeping allowance ($s$) he will give spouse $f$. It would be more realistic to let spouses to commit to a binding allowance ($t$) and then allow the husband to choose the supplementary allowance ($s$) as in Lundberg and Pollak (1993), because it is unlikely that the husband in this household resource management arrangement would not give any money to his wife. However, the results do not differ and it simplifies the proofs. One way to reconcile this idea is to think about the problem in terms of spouses' disposable income, net of the binding allowance ($t$).
and in stage three, spouse \( f \) decides the public good provision conditional of both \( T \) and \( s \). The model is solved by backwards induction.

In particular, spouse \( f \) solves the following optimization problem,

\[
\max_{Q \geq 0; x_f \geq 0} U_f = v(Q) + u(x_f) \quad \text{s.t.} \quad x_f \leq Y_f + s - pQ
\]

(13)

Substituting in the budget constraint, the First-Order condition for \( Q \) is

\[
v'(Q) - pu'(Y_f + s - pQ) \leq 0
\]

(14)

Conducting comparative statics on the above condition yields,

\[
\frac{\partial Q}{\partial s} = \frac{pu'(Y_f + s - pQ)}{v'(Q) + pu'(Y_f + s - pQ)} > 0
\]

(15)

So, the housekeeping allowance is the husband’s way to increase his household good consumption, but the correspondence is not one-to-one. Note that, the public good allocation will be strictly positive, thus equation (14) holds with equality.

Taking spouse \( f \)'s first-order condition as given, spouse \( m \) solves:

\[
\max_{s \geq 0; x_m \geq 0; Q \geq 0} U_m = v(Q) + u(x_m) \quad \text{s.t.} \quad x_m \leq Y_m - s; \quad v'(Q) - pu'(Y_f + s - pQ) = 0
\]

(16)

The Lagrangian is:

\[
\mathcal{L} = v(Q) + u(Y_m - s) + \lambda [pu'(Y_f + s - pQ) - v'(Q)]
\]

which yields the following Kuhn-Tucker first-order conditions,

\[
\frac{\partial \mathcal{L}}{\partial Q} = v'(Q) - \lambda p^2 u''(Y_f + s - pQ) - \lambda v''(Q) \leq 0
\]

(17)

\[
\frac{\partial \mathcal{L}}{\partial s} = -u'(Y_m - s) + \lambda pu''(Y_f + s - pQ) \leq 0
\]

(18)

\[5\] This is equivalent to setting the optimization problem in the following way:

\[
\max_{s \geq 0; x_m \geq 0; Q \geq 0} U_m = v(Q(s)) + u(x_m) \quad \text{s.t.} \quad x_m \leq Y_m - s; \quad Q(s) \geq 0
\]

And then using equation (15) in the FOC to substitute for \( \frac{\partial Q}{\partial s} \).
\[ \frac{\partial c}{\partial \lambda} = pu'(Y_f + s - pQ) - v'(Q) = 0 \]  
\[ Q \left[ \frac{\partial c}{\partial Q} \right] = 0, s \left[ \frac{\partial c}{\partial s} \right] = 0; \lambda \left[ \frac{\partial c}{\partial \lambda} \right] = 0; Q \geq 0; s \geq 0 \]  

Solving the system of first-order conditions simultaneously yields the Subgame Perfect Nash equilibrium. There is a corner solution where the housekeeping allowance can be non-positive, as well as an interior solution. Proposition 5 specifies the conditions that must be met for an interior solution to exist.

**Proposition 5:** Given \( Y_f \), there exists a \( \bar{Y}_m \) in the interval \((0, Y_f)\) such that the Subgame Perfect Nash equilibrium is a corner solution with \( s = 0 \) and \( Q > 0 \) if \( Y_m \leq \bar{Y}_m \).

As in the independent management model, the properties of the corner solution equilibrium are different from the properties of the interior equilibrium; whether the housekeeping allowance occurs depends on how \( m's \) income compares to \( f's \) income. These properties are demonstrated in Proposition 6 and 7, focusing on the implications for changes in \( f's \) income.

**5.1 Corner Equilibrium:**

Following Proposition 5, if \( Y_m \leq \bar{Y}_m \in (0, Y_f) \), it is optimal for \( m \) to give a non-positive housekeeping allowance to \( f \). As shown by Lundberg and Pollak (1993), this yields an inefficient outcome that could be improved upon by bargaining. Proposition 6 states the properties of the equilibrium with respect to changes in \( f's \) income and provides the foundations as to why in corner equilibrium there are no incentives to hide a monetary transfer from \( m \).
**Proposition 6:** In a corner solution,

If \( Y_m \leq \bar{Y}_m \in (0,Y_f) \) thus \( s = 0 \),

An increase in \( Y \) results in \( \frac{\partial x_f}{\partial Y_f} > 0 \); \( \frac{\partial q}{\partial Y_f} > 0 \); \( \frac{\partial s}{\partial Y_f} = \frac{\partial x_m}{\partial Y_f} = 0 \), while an increase in \( Y_m \) results in

\[ \frac{\partial x_m}{\partial Y_m} > 0; \frac{\partial s}{\partial Y_m} = 0; \frac{\partial q}{\partial Y_m} = \frac{\partial x_f}{\partial Y_m} = 0. \]

If spouse \( m \) is not making a positive housekeeping allowance to \( f \), changes in \( Y_f \) have no impact on \( m \)'s allocations. Now consider the case when \( f \) receives a transfer \( (T) \) that is observable to household member \( m \) with probability zero. Spouse \( f \) then has to decide whether to allocate the monetary transfer \( (T) \) between private and household good consumption, thus directly or indirectly informing \( m \) about the increase in her resources, or to hide it and spend it all on private consumption. If the distribution of income is such that \( Y_m \leq \bar{Y}_m \in (0,Y_f) \), then there is no incentive to hide the transfer because a change in \( Y_f \) only impacts \( f \)'s allocations\(^6\).

### 5.2 Housekeeping Allowance Equilibrium

The case where \( m \) gives a housekeeping allowance to his wife is efficient within the context of a housekeeping allowance management system in which spouses do not have access to all of the household resources, and is self-enforcing because no binding agreements are made. In this case, as long as \( Y_m > \bar{Y}_m \), it is \( m \)'s best response to give a strictly positive housekeeping allowance to \( f \) in order to increase his household good consumption. Proposition 7 states the properties of the housekeeping allowance equilibrium.

---

\(^6\) As in the independent management case, there exists another case that is not being examined in this paper, corresponding to when the transfer is such that, if revealed, it makes the interior equilibrium possible.
Proposition 7 In a housekeeping allowance equilibrium, if \( Y_m > \overline{Y_m} \), thus \( S, Q > 0 \), an increase in \( Y_f \) results in

\[
\frac{\partial x_f}{\partial Y_f} > 0; \quad \frac{\partial Q_f}{\partial Y_f} > 0; \quad \frac{\partial x_m}{\partial Y_f} > 0; \quad \frac{\partial S}{\partial Y_f} < 0.
\]

When \( m \) gives a strictly positive housekeeping allowance to his wife, an increase in \( Y_f \) increases both \( f \) and \( m \)'s private consumption and \( f \)'s contribution to the public good, though it is likely to decrease \( m \)'s supplementary transfer. This is the source of the incentive to hide income. If \( f \) reveals that her resources have increased, in order to increase her public good consumption, she will first have to compensate the reduction in spouse \( m \)'s housekeeping allowance, and then supplement her private and household good consumption. If she hides however, she can keep her household good consumption unchanged by preventing \( m \) from reducing his allowance, and increase her private consumption in the amount of the transfer.

Now consider the case when \( f \) receives a transfer \((T)\) and has to decide whether to allocate \( T \) between private and household good consumption, or to hide it and spend it all on private consumption. If the conditions described in Proposition 8 are met, \( f \) will hide the transfer from \( m \).

Proposition 8: Given \( Y_f, Y_m \) when \( Y_m > \overline{Y_m} \), there exists a threshold level of transfer \((\overline{T})\) such that for any \( T < \overline{T} \) the Subgame Perfect Nash Equilibrium of the game is to hide the transfer.

As before, if the change in utility per unit change in the transfer is higher when \( f \) hides the transfer compared to when she reveals it, income-hiding is an equilibrium. The decision to hide depends not only on the relative change in marginal utility of private and public consumption for both household members, but on the size of the transfer as well, such that small transfers will be hidden. So far, I have shown that when spouses have independent accounts or when there is gender specialization,
there is a threshold level of transfer such that income hiding is an equilibrium. However, without assuming a specific functional form, I cannot determine if hiding is equally likely in both management systems. The Cobb-Douglas example provides further intuition on the decision to hide money, and indicates that, ceteris paribus, a household that adopts a housekeeping allowance system is more likely to hide than a household with independent accounts.

5.3 Illustrative Example:

Consider preferences are of the form specified in (7), then spouse $f$ solves:

$$\max_{Q \geq 0; x_f \geq 0} U_f = \alpha_f \log(x_f) + (1 - \alpha_f) \log(Q) \quad \text{s.t.} \quad x_f \leq Y_f + s - pQ$$

which yields the following first order condition:

$$(1 - \alpha_f)(Y_f + s) \leq pQ$$ \hspace{1cm} (21)

From (21) we can see that it holds with equality given that $Q$ is never going to be equal to zero.

Taking spouse $f$'s first order condition (21) as given and substituting it in for $Q$, spouse $m$ solves:

$$\max_{s \geq 0; x_f \geq 0} U_m = \alpha_m \log(x_m) + (1 - \alpha_m) \log\left(\frac{(1 - \alpha_f)(Y_f + s)}{p}\right) \quad \text{s.t.} \quad x_m \leq Y_m - s$$ \hspace{1cm} (22)

The Kuhn-Tucker first-order condition for $s$ yields the following,

$$(1 - \alpha_m)Y_m - \alpha_m Y_f \leq s$$ \hspace{1cm} (23)

According to (23) there exists the possibility of a non-positive housekeeping allowance.

(i) Corner Equilibrium

From condition (23) we can see that if $(1 - \alpha_m)Y_m \leq \alpha_m Y_f$ then $s=0$, and thus $Q = \frac{(1 - \alpha_f)(Y_f)}{p}$, $x_f = \alpha_f(Y_f)$, $x_m = Y_m - t$. Note that a monetary transfer, has no impact on the housekeeping
allowance, and thus no impact on \(m\)'s allocations, so there are no incentives for \(f\) to hide if this is the relevant equilibrium.

(ii) \textbf{Housekeeping Allowance Equilibrium}

Following from condition (23), if \((1 - \alpha_m)Y_m > \alpha_m Y_f\) then \(s = (1 - \alpha_m)Y_m - \alpha_m Y_f\), \(x_m = \alpha_m(Y_f + Y_m)\), \(Q = \frac{(1-\alpha_f)(1-\alpha_m)}{p}(Y_f + Y_m)\), \(x_f = \alpha_f(1 - \alpha_m)(Y_f + Y_m)\).

It is clear that if spouse \(f\) receives a monetary transfer \(T\), \(m\)'s supplementary transfer decreases in the amount \(-\alpha_m\) per unit of transfer, so \(f\) is faced with a trade-off that will determine her decision to reveal. If she reveals \(T\) the equilibrium allocations are:

\[ s = (1 - \alpha_m)Y_m - \alpha_m(Y_f + T), \quad x_m = \alpha_m(Y_f + Y_m + T), \]
\[ Q = \frac{(1-\alpha_f)(1-\alpha_m)}{p}(Y_f + Y_m + T), \quad x_f = \alpha_f(1 - \alpha_m)(Y_f + Y_m + T). \]

Thus, the increase in \(f\)'s resources increases public good consumption and private expenditure relative to the allocations before the transfer, though private expenditure does not increase as much because she first has to compensate for the reduction in \(m\)'s housekeeping allowance. On the other hand, if \(f\) hides \(T\), \(Q\) and \(s\) remain unchanged, but \(f\)'s private consumption increases in the amount of the transfer, such that \(x_f = \alpha_f(1 - \alpha_m)(Y_f + Y_m) + T\).

Following Proposition 8, the decision to reveal is given by comparing the change in utility per unit of \(T\) given spouse \(f\) chooses to reveal versus is she prefers to hide the money. So, \(f\) hides the transfer from \(m\) if and only if,

\[
\left. \frac{\partial U_f}{\partial T} \right|_R = \frac{\partial v}{\partial Q} \frac{\partial Q}{\partial T} + \frac{\partial u}{\partial x_f} \frac{\partial x_f}{\partial T} = \frac{1}{Y_f + T + Y_m} < \frac{\alpha_f}{\alpha_f(1 - \alpha_m)(Y_f + Y_m) + T} = \left. \frac{\partial U_f}{\partial T} \right|_{NR} \tag{24}
\]

Re-arranging yields \(\alpha_f(1 - \alpha_m)(Y_f + Y_m) + T < \alpha_f(Y_f + T + Y_m)\) and the condition simplifies to \(\frac{\alpha_f(1 - \alpha_m)(Y_f + Y_m)}{(1 - \alpha_f)} < T\). The condition to hide in the independent management case depends on the
size of the transfer as well, but the threshold level of transfer required to induce revelation is smaller in the independent management case. Thus hiding is more likely to occur in a housekeeping allowance resource management system, relative to households under independent management.

6. Collective Bargaining Model

In this section, I describe the equilibrium allocations that result when spouses bargain over public and private consumption collectively, which corresponds to the case where spouses adopt a joint resource management system. In this type of household, both spouses have access to all of the household resources and jointly decide how to allocate them. I model this case using an augmented version of the Browning and Chiappori (1998) collective bargaining model, which is more general relative to other bargaining models that assume specific threat points and functional forms. In this version of the collective household model, everything but the monetary transfer received by \( f \) is common knowledge, such that the decision to hide can be thought of as the step prior to choosing to pool income every time new resources are available to each household member.

The information asymmetry is introduced by allowing the monetary transfer \( (T) \) received by spouse \( f \) to be observable with probability zero. In deciding to reveal or hide the transfer, \( f \) faces a trade-off between increasing her own discretionary spending and increasing her bargaining power. If she hides the transfer, \( f \) may spend the entire amount without influence from her spouse. But, public goods are observable by both spouses. Therefore, if the wife is to successfully hide her additional income, she can spend it only on private consumption, which is unobservable. Conversely, if she reveals the transfer, the wife can increase her influence over intra-household allocation decisions, but her income will effectively be taxed via the bargaining process. I allow
household members to have different preferences, bargain over all allocations, and assume they can negotiate binding agreements with zero transaction costs.

The collective bargaining equilibrium is solved in two stages. In the first stage, \( f \) receives a monetary transfer \( T \) and decides whether or not to reveal it. Given that this is a cooperative setting, revelation in this case is equivalent to pooling. In the second stage, \( f \) and \( m \) bargain over the public good allocation and the share of the remaining resources that each will get for their private consumption. These resources are divided according to a sharing rule that depends on each spouse's bargaining power. The game is solved by backwards induction, so first I find the optimal public good allocation and private expenditure shares conditional on the amount of the transfer that is revealed, and then derive the conditions that must be met for \( f \) to reveal the transfer.

In the second stage, the objective function of the collective household is the bargaining power weighted sum of each member’s utility:

\[
C = \mu(u(x_i) + v(Q)) + (1 - \mu)(u(x_j) + v(Q))
\]  

(25)

Where \( \mu \) is the relative bargaining power of spouse \( f \) and \( (1 - \mu) \) is the bargaining power of spouse \( m \). This is the weight given to each spouse’s utility in the household welfare function when bargaining, and it is partially determined by each spouse’s outside options \( (U_a, U_b) \), as well as by resources originally brought into the marriage, and distribution factors such as culture and law \( (z) \). At this point I will not assume any threat point in particular because the goal is to model the decision to hide income by a spouse in a joint management system. Therefore, I use the bargaining weight as a generic way to incorporate the existence of a threat point, such that \( \mu = g(Y_f, Y_m, z, p) \). Consistent with both non-cooperative equilibria within marriage and divorce threat points, I assume the transfer increases \( f’s \) bargaining power \( (\mu) \). However, I do not specify a functional form in order to avoid making assumptions about the relative weights resources would have over other factors that influence bargaining power, but that do not vary when the quantity of resources increases. In the
following section, I elaborate on how non-cooperative threat points within marriage would influence bargaining power when a transfer occurs.

The collective household’s problem is to maximize (25) subject to the aggregate budget constraint (1). Given that households with joint resource management systems have already found a way to cooperate and negotiate binding contracts, I solve the collective model assuming that the participation constraints do not bind, i.e. assuming that both spouses are better off cooperating than under the threat points. This is not a strong assumption given that spouses are bargaining over all allocations, such that the public good provision will be efficient (at least when all information is revealed).

\[
\max_{Q,x_f \geq 0} \mu \{u(x_f) + v(Q)\} + (1 - \mu) \{u(Y_f + Y_m + T - x_f - pQ) + v(Q)\} \tag{26}
\]

where \( Y = Y_f + Y_m \). For now assume \( T=0 \). The Kuhn-Tucker first-order conditions of the problem in (26) are:

\[
\frac{\partial c}{\partial Q} = v'(Q) - (1 - \mu)pu'\left(Y_f + Y_m - x_f - pQ\right) \leq 0
\]

\[
\frac{\partial c}{\partial x_f} = \mu u'(x_f) - (1 - \mu)u'\left(Y_f + Y_m - x_f - pQ\right) \leq 0
\]

\[
Q \left[ \frac{\partial c}{\partial Q} \right] = 0; \ x_f \left[ \frac{\partial c}{\partial x_f} \right] = 0; \ Q, x_f \geq 0 \tag{27}
\]

Solving this system yields the demand for the household public good and the demand for resources for private consumption. The Kuhn Tucker conditions imply that in this case, there are no corner solutions. The optimal demands respond to changes in aggregate income (i.e. income pooling feature) and to changes in individual income through bargaining power. Proposition 9 states the comparative statics.
Proposition 9: An increase in aggregate income (Y) increases public and private consumption for both household members, whereas an increase in f’s bargaining power (μ) increases her private consumption, decreases m’s private consumption, and can either increase or decrease public good consumption.

In the first stage, f must decide whether to reveal the transfer or to hide it from m, thus f compares for each case the change in utility per unit of T. Proposition 9 implies that if f receives a monetary transfer (T) she faces the following trade-off: if she hides the transfer, she can get more private expenditure relative to the case where she reveals and pools all of her resources. If she reveals, f can increase her household good consumption and her bargaining power, but both her private and public good consumption will not increase as much as she would like because they are effectively taxed by bargaining power.

Proposition 10: Given Y_f, Y_m and T, there exists a strictly positive threshold change in bargaining power Δμ such that for any Δμ < Δμ income-hiding is the Subgame Perfect Nash Equilibrium if:

\[ p^2μ(1 - μ)u'(x_f)u''(x_m)[u'(x_f) - μu'(x_f)] + (1 - μ)v'(Q)u''(x_m)[u'(x_f) - u'(x_f)] + μu'(x_f)v''(Q)u''(x_f) > 0 \] (28)

Corollary 1: Given Y_f, Y_m and T, Δμ is decreasing in the initial level of bargaining power of spouse f, μ, such that as μ approaches zero, the threshold level of bargaining power is strictly negative, whereas when μ tends to 1 it is positive.

Proposition 10 implies that the decision to hide money when bargaining depends not only on the change in bargaining power but on the initial level of bargaining power as well. Corollary 1 implies
that if \( f \) has low bargaining power to begin with, the threshold level of change in bargaining power below which she will hide the transfer from \( m \) is lower, whereas as \( f \)'s bargaining power approaches 1 she will be more likely to withhold money. This result is intuitive because if \( f \)'s bargaining power is low, she is less likely to influence household allocations towards her preferences and thus her private consumption is “taxed” more severely, and thus any improvement in her bargaining position makes her better off. When bargaining power is high, the public good allocation will be closer to her preferences and thus, in the margin, the benefit from cooperating is much lower.

### 6.1 Illustrative Example

In the first stage, the optimization problem of the collective household is thus to maximize the bargaining power weighted sum of individual utility functions in (7) subject to the household (or aggregate) budget constraint (1) conditional on \( T \).

\[
\max_{q, x_f \geq 0} \mu \left\{ \alpha_f \log(x_f) + (1 - \alpha_f) \log(q) \right\} + (1 - \mu) \left\{ \alpha_m \log(Y - x_f - pQ) + (1 - \alpha_m) \log(Q) \right\}
\]

Recall that \( Y = Y_f + Y_m + T \). The Kuhn-Tucker first-order conditions are:

\[
\frac{\partial c}{\partial Q} = \left[ \mu (1 - \alpha_f) + (1 - \mu)(1 - \alpha_m) \right] \left[ Y - x_f \right] - \left[ \mu (1 - \alpha_f) + (1 - \mu) \right] pQ \leq 0
\]

\[
\frac{\partial c}{\partial x_f} = \mu \alpha_f \left[ Y - pQ \right] - \left[ \mu \alpha_f + (1 - \mu) \alpha_m \right] x_f \leq 0
\]

Solving the system yields the optimal demands if the transfer is revealed, where \( \bar{\mu} \) is the new level of bargaining power given \( T \), \( \bar{\mu} = \mu(T > 0) \) and \( \mu = \mu(T = 0) \).

\[
Q^{B*} = \left[ \bar{\mu} (1 - \alpha_f) + (1 - \bar{\mu})(1 - \alpha_m) \right] \left[ \frac{Y_f + Y_m + T}{p} \right]
\]
\[ x_f^{B*} = \bar{m} \alpha_f [Y_f + Y_m + T] ; \quad x_m^{B*} = (1 - \bar{m}) \alpha_m [Y_f + Y_m + T] \]

If \( f \) hides the transfer however,

\[ Q^{BH*} = \left[ \mu (1 - \alpha_f) + (1 - \mu)(1 - \alpha_m) \right] \frac{[Y_f + Y_m]}{p} \quad (31.a) \]

\[ x_f^{BH*} = \mu \alpha_f [Y_f + Y_m] + T ; \quad x_m^{BH*} = (1 - \mu) \alpha_m [Y_f + Y_m] \]

The decision to hide the transfer follows from the condition stated in Proposition 11, such that \( f \) hides iff

\[ \frac{\alpha_f}{\mu \alpha_f (Y_f + Y_m) + T} = \frac{\partial U_f}{\partial T} \bigg|_{NR} \]

(32)

Solving for the threshold change in bargaining power,

\[ \frac{\partial \mu}{\partial T} < \left[ \frac{\bar{\mu} \left[ \bar{\mu} (1 - \alpha_f) + (1 - \bar{\mu})(1 - \alpha_m) \right]}{\bar{\mu} (\alpha_m - \alpha_f) + (1 - \alpha_m) \alpha_f} \right] \frac{\alpha_f}{\mu \alpha_f (Y_f + Y_m) + T} - \frac{1}{Y_f + T + Y_m} \]

Note that

\[ \frac{\alpha_f}{\mu \alpha_f (Y_f + Y_m) + T} - \frac{1}{Y_f + T + Y_m} = \frac{\alpha_f (1 - \mu)[Y_f + Y_m] - (1 - \alpha_f)T}{\mu \alpha_f (Y_f + Y_m) + T} \left[ \frac{1}{Y_f + T + Y_m} \right] > 0 \iff \frac{\alpha_f (1 - \mu)[Y_f + Y_m]}{(1 - \alpha_f)} > T \]

and

\[ \frac{\bar{\mu} [\bar{\mu} (1 - \alpha_f) + (1 - \bar{\mu})(1 - \alpha_m)]}{\bar{\mu} (\alpha_m - \alpha_f) + (1 - \alpha_m) \alpha_f} > 0 \]

so, if the transfer is small enough relative to aggregate household income, \( f \) has an incentive to hide money from \( m \) because the change in bargaining power is not enough to compensate for the loss in private consumption.

Finally, if we compare the threshold levels of transfer such that hiding is the equilibrium for the three different management systems considered in this paper, we can see that:

\[ \frac{\alpha_f \alpha_m [Y_f + Y_m]}{(1 - \alpha_f)} > \frac{\alpha_f \alpha_m [Y_f + Y_m]}{(1 - \alpha_f \alpha_m)} > \frac{\alpha_f (1 - \mu)[Y_f + Y_m]}{(1 - \alpha_f)} > T \quad (33) \]
So hiding is more likely to occur in the housekeeping allowance case, relative to both, an independent and a joint management household. As long as spouse $m$ has no dictatorial power, it is not clear whether hiding is more likely to occur under an independent management contract, compared to a pooling household. In general, households under independent management will be more prone to hiding; if we let $a_m = 1 - \mu$ then it is clear that an independent management contract would be more prone to hiding; and if $a_m < 1 - \mu$ then (33) holds as well. The only case where it is not clear is when $a_m > 1 - \mu$.

7. Conclusions

In this paper I presented three models of income-hiding between household members when one spouse has an information advantage regarding the quantity of resources available to the household. I show the conditions that must be met for income hiding to occur in equilibrium, and find that income hiding is more likely to occur under certain contractual arrangements than others depending on who has control over resources. In general, my results indicate that income-hiding is more likely to occur in a household with a housekeeping allowance contract, relative to both, an independent management and a joint management household. A collective household is the least likely to observe hiding, as long as the spouse without the information advantage has no dictatorial power.

In equilibrium, a spouse with an information advantage in a collective household chooses to hide the transfer if the change in bargaining power associated with informing her spouse is not significant enough to compensate for the loss in discretionary expenditure that results from the bargaining process. In particular, there exists a strictly positive threshold of change in bargaining power needed to induce revelation, and the threshold is decreasing on the wife’s initial power.
8. Appendix: Proofs

8.1 Proof of Proposition 1:

Equation (5) implies that $Q_f = 0$ for some $Q_m > 0$ as long as
\[ v'(Q_m) < p u'(Y_f) \]  \hfill (P1.1)

If $Y_f = 0$ (P1.1) holds because it was assumed that both goods are normal, $u'(0) = \infty$. If $Y_f = Y_m$, from the concavity assumption it follows that $u'(Y_m) < u'(Y_m - pQ_m)$. But equation (6) implies that $v'(Q_m) = P u'(Y_m - pQ_m)$ therefore:
\[ u'(Y_m) < u'(Y_m - pQ_m) = v'(Q_m) \]  \hfill (P1.2)

Furthermore $u(Y_f)$ is increasing in $Y_f$. Therefore there exists a unique $Y_f \in (0, Y_m)$. Likewise, since the problem is symmetric there exists a unique $Y_m \in (0, Y_f)$. 

8.2 Proof of Proposition 2:

Given that the problem is symmetric, it suffices to derive the comparative statics only for a change in $Y_f$ which is also the comparative statistic of interest for the propositions that follow.

Case (i): If $Y_f \leq Y_m \in (0, Y_m)$ thus $Q_f = 0$ and $x_f = Y_f$ so the value of $Q_m$ is obtained from spouse m’s optimization condition (3).
\[ u'(Y_m - pQ_m) = v'(Q_m) \]  \hfill (P2.1)

Note that $Q_m$ does not depend on $Y_f$ and neither does $\alpha_m$, therefore the only variable that changes with $Y_f$ is $x_f$.

\[ \frac{\partial Q_f}{\partial Y_f} = 0 \]  \hfill (P2.3)
\[ \frac{\partial x_f}{\partial Y_f} = 1 \]  \hfill (P2.4)
\[ \frac{\partial Q_m}{\partial Y_f} = 0 \]  \hfill (P2.5)
\[ \frac{\partial x_m}{\partial Y_f} = 0 \]  \hfill (P2.6)
\[ \frac{\partial Q_f}{\partial Y_m} = 0 \]  \hfill (P2.3)
\[ \frac{\partial x_f}{\partial Y_m} = 0 \]  \hfill (P2.4)
\[ \frac{\partial Q_m}{\partial Y_m} = \frac{pu'r(x_m)}{vr(Q_m) + p^2 u'r(x_m)} \]  \hfill (P2.5)
\[ \frac{\partial x_m}{\partial Y_m} = \frac{pu'r(Q_m)}{vr(Q_m) + p^2 u'r(x_m)} \]  \hfill (P2.6)

Case (ii): If $Y_m \leq Y_m \in (0, Y_f)$ thus $Q_m = 0$, so the value of $Q_f$ is obtained from spouse f’s optimization condition (6)
\[ pu'(Y_f - pQ_f) = v'(Q_f) \]  \hfill (P2.2)

Differentiating (P2.2) and f’s budget constraint with respect to $Y_f$ yields the results stated in the proposition. Note that neither $\alpha_m$ nor $Q_m$ change with $Y_f$. In particular,

\[ \frac{\partial Q_f}{\partial Y_f} = \frac{pu'r(x_f)}{vr(Q_f) + p^2 u'r(x_f)} > 0 \]  \hfill (P2.3)
\[ \frac{\partial x_f}{\partial Y_f} = \frac{vr(Q_f)}{vr(Q_f) + p^2 u'r(x_f)} > 0 \]  \hfill (P2.4)
\[ \frac{\partial Q_m}{\partial Y_f} = 0 \]  \hfill (P2.5)
\[
\frac{\partial x_m}{\partial y_f} = 0 \quad (P2.6)
\]
\[
\frac{\partial q_f}{\partial y_m} = 0 \quad (P2.3)
\]
\[
\frac{\partial q_f}{\partial y_f} = 0 \quad (P2.4)
\]
\[
\frac{\partial q_m}{\partial y_m} = 0 \quad (P2.5)
\]
\[
\frac{\partial x_m}{\partial y_m} = 1 \quad (P2.6)
\]

8.3 **Proof of Proposition 3:**

Given that the problem is symmetric, it suffices to derive the comparative statics only for a change in \( Y_f \) which is also the comparative statistic of interest for the propositions that follow. If \( Y_f > Y_f^* \) and \( Y_m > Y_m^* \), then \( q_m, q_f > 0 \), and the equilibrium allocations are obtained by solving the following system for \( q_m \) and \( q_f \):

\[
u'(Y_f - p q_f) = \nu'(Y_m - p q_m) = \nu'(q_f + q_m)
\]

The results in the proposition are obtained from totally differentiating the system in \((P3.1)\). In matrix form:

\[
\begin{bmatrix}
\nu''(Q) + p^2 u''(x_m) & \nu''(Q) \\
\nu''(Q) & \nu''(Q) + p^2 u''(x_f)
\end{bmatrix}
\begin{bmatrix}
\frac{d q_m}{d y_m} \\
\frac{d q_f}{d y_f}
\end{bmatrix}
= \begin{bmatrix}
u''(x_m) & 0 \\
0 & \nu''(x_f)
\end{bmatrix}
\begin{bmatrix}
\frac{d y_m}{d y_f}
\end{bmatrix}
\]

Let \( J \) denote determinant of the Hessian which is equal to:

\[
J = \text{det} \begin{bmatrix}
\nu''(Q) + p^2 u''(x_m) & \nu''(Q) \\
\nu''(Q) & \nu''(Q) + p^2 u''(x_f)
\end{bmatrix}
= p^2 \{ \nu''(Q) [u''(x_f) + u''(x_m)] + p^2 u''(x_f) u''(x_m) \} > 0
\]

The comparative statics are given by:

\[
\frac{\partial q_f}{\partial y_f} = \frac{p u''(x_f) u''(x_m)}{J} > 0 \quad (P3.3)
\]
\[
\frac{\partial q_f}{\partial y_m} = -\frac{p u''(x_f) u''(x_m)}{J} < 0 \quad (P3.3)
\]
\[
\frac{\partial x_f}{\partial y_f} = \frac{p^2 u''(x_m)}{J} > 0 \quad (P3.4)
\]
\[
\frac{\partial x_f}{\partial y_m} = \frac{p u''(x_f) u''(x_m)}{J} > 0 \quad (P3.4)
\]
\[
\frac{\partial q_m}{\partial y_m} = \frac{p u''(x_f) u''(x_m) + p^2 u''(x_f) u''(x_m)}{J} > 0 \quad (P3.3)
\]
\[
\frac{\partial q_m}{\partial y_f} = -\frac{p u''(x_f) u''(x_m)}{J} < 0 \quad (P3.5)
\]
\[
\frac{\partial x_m}{\partial y_m} = \frac{p^2 u''(x_f) u''(x_f)}{J} > 0 \quad (P3.6)
\]
\[
\frac{\partial x_m}{\partial y_f} = \frac{p^2 u''(x_f) u''(x_f)}{J} > 0 \quad (P3.6)
\]

8.4 **Proof of Proposition 4:**

Assumptions:
(i) Spouse \(m\) can observe the transfer with probability zero.

(ii) The household public good is perfectly observable, such that spouse \(m\) can infer that a transfer has occurred and thus adjust his contribution accordingly.

(iii) Spouse \(f\)'s private consumption, or discretionary expenditure, is not monitored by \(m\).

If \(f\) chooses to reveal the transfer and \(Y_f + T > \bar{Y}_f\) and \(Y_m > \bar{Y}_m\) (i.e. both spouses are making positive contributions to the public good) the demands are obtained from solving the following system of equations:

\[
p u'(Y_f + T - pQ_f) = p u'(Y_m - pQ_m) = v'(Q_f + Q_m) \quad (P4.1)
\]

If \(f\) receives a transfer \(T\) and decides to reveal it, the change in \(Q_f, Q_m, x_f, x_m\) per unit change in \(T\) are equivalent to those corresponding to changes in \(Y\), described in Proposition 2.

The change in utility per unit change in the transfer is given by:

\[
\frac{\partial u_f}{\partial T} \bigg| R = \frac{\partial u_f}{\partial q} \left[ \frac{\partial q_f}{\partial T} + \frac{\partial q_m}{\partial T} \right] + \frac{\partial u_f}{\partial x_f} \frac{\partial x_f}{\partial T} + \frac{w(q)}{f} \left[ p^3 u''(\bar{x}_f) u'''(\bar{x}_m) \right] + \frac{w(q)}{f} \left[ p^2 v''(Q) u''(\bar{x}_m) \right] \quad (P4.2)
\]

If \(f\) decides to hide the transfer and \(Y_f + T > \bar{Y}_f\) and \(Y_m > \bar{Y}_m\), then \(f\) spends all the transfer on private consumption and \(f\)'s household good contribution doesn't change compared to before the transfer, nor do \(m\)'s allocations. So it must be that \(u(x_f) < u(\bar{x}_f) < u(\bar{x}_f)\) where \(\bar{x}_f = x_f + T\) where \(x_f\) is the pre-transfer private consumption optimal allocation and \(\bar{x}_f\) is the post-transfer private consumption optimal allocation if the transfer is revealed.

The change in utility per unit change in the transfer is given by:

\[
\frac{\partial u_f}{\partial T} \bigg| NR = u'(\bar{x}_f) \quad (P4.3)
\]

Spouse \(f\) hides \(T\) from \(m\) if and only if:

\[
\frac{\partial u_f}{\partial T} \bigg| R = \frac{w(q)}{f} \left[ p^3 u''(\bar{x}_f) u'''(\bar{x}_m) \right] + \frac{w(q)}{f} \left[ p^2 v''(Q) u''(\bar{x}_m) \right] < u'(\bar{x}_f) = \frac{\partial u_f}{\partial T} \bigg| NR \quad (P4.4)
\]

Simplifying the above expression:

\[
p^2 u''(\bar{x}_f) u'''(\bar{x}_m) \left[ v'(Q) - p u'(\bar{x}_f) \right] + p v''(Q) u''(\bar{x}_m) \left[ u'(\bar{x}_f) - u'(\bar{x}_f) \right] < p u'(\bar{x}_f) v''(Q) u''(\bar{x}_f) \quad (P4.5)
\]

Taking into account \(f\)'s FOC \(p u'(\bar{x}_f) = v'(Q)\), \(f\) hides if:

\[
\left[ u'(\bar{x}_f) - u'(\bar{x}_f) \right] \left[ p^2 u''(\bar{x}_f) u'''(\bar{x}_m) + p v''(Q) u''(\bar{x}_m) \right] < u'(\bar{x}_f) v''(Q) u''(\bar{x}_f) \quad (P4.6)
\]

This yields the condition that must be met for \(f\) to hide the transfer, and it can be re-written as:

\[
\left[ u'(\bar{x}_f) - u'(\bar{x}_f) \right] \left[ p^2 \frac{w(x_m)}{w(q)} + \frac{w(x_m)}{w(q)} \right] < u'(\bar{x}_f) \quad (P4.7)
\]

The left hand side of \(P4.7\) is positive due to the concavity assumption \(u'(\bar{x}_f) > u'(\bar{x}_f)\) because \(\bar{x}_f < \bar{x}_f\). Such that, the decision to hide depends on the relative change in marginal utility of private and public consumption for both household members, and on the size of the transfer.

Consider the extreme case where \(f\) doesn’t hide and allocates all of the transfer towards the household public good, such that \(T = pQ_f\). As the transfer increases \((T \to \infty)\), \(\lim_{T \to \infty} u'(\bar{x}_f) = \lim_{T \to \infty} u'(\bar{x}_f) = \lim_{T \to \infty} u'(Y_f + T - pQ_f) = \lim_{T \to \infty} u'(Y_f + T - T) = u'(Y_f)\). If she does hide, then her only option is to allocate it towards private consumption to avoid detection, thus \(\lim_{T \to \infty} u'(\bar{x}_f) = \lim_{T \to \infty} u'(x_f + T) = u'(\infty) \to 0\). The right-hand side of \(P4.7\) is positive and the left-hand side tends to zero (cannot be equal to zero by assumption), so in this case revealing would be the equilibrium.

Now consider the other extreme case where the transfer tends to zero. If \(f\) reveals the transfer: \(\lim_{T \to 0} u'(\bar{x}_f) = \lim_{T \to 0} u'(Y_f + T - pQ_f) = u'(x_f)\), if she hides it \(\lim_{T \to 0} u'(\bar{x}_f) = \lim_{T \to 0} u'(x_f)\).
lim_{T \to 0} u'(x_f + T) = u'(x_f), so (P4.7) simplifies to 0 < u'(x_f). Thus there exists a threshold level of transfer ($\bar{T}$) such that for any $T < \bar{T}$ the Subgame Perfect Nash Equilibrium is to hide.

8.5 Proof of Proposition 5:
First, it is important to show that (14) binds. Let $Q=0$, then (14) implies:

$$v'(0) < pu'(Y_f + s) \quad (P5.1)$$

But by assumption $v'(0) = \infty$, so (14) binds and $Q>0$.
Equation (18) implies that $s=0$ for some $Q>0$ as long as:

$$\lambda pu''(Y_f - p Q) < u'(Y_m) \quad (P5.2)$$

Which only holds iff $\lambda < 0$. We have shown that (14) binds, therefore the constraint on $m$'s problem binds as well, so $\lambda \neq 0$. Since $Q>0$, from (17) we know:

$$v'(Q) = \lambda[p^2 u''(Y_f - p Q) + v''(Q)] \quad (P5.3)$$

Which, given the concavity assumption, is only possible if $\lambda < 0$.
If $Y_m = 0$, (P5.2) holds because $u'(0) = \infty$.

$$\lambda pu''(Y_f - p Q) < u'(0) \quad (P5.4)$$
If $Y_m = Y_f$, due to the concavity assumption we know that $u''(Y_f - p Q) > u'(Y_f)$, and from (14) and (17) we know that:

$$pu'(Y_f - p Q) = v'(Q) = \lambda[p^2 u''(Y_f - p Q) + v''(Q)] \quad (P5.5)$$

So,

$$pu'(Y_f) < pu'(Y_f - p Q) = \lambda[p^2 u''(Y_f - p Q) + v''(Q)] \quad (P5.6)$$

So, following from (18), and multiplying (P5.4) by $p$ on both sides:

$$\lambda p^2 u''(Y_f - p Q) < pu'(Y_f) < \lambda[p^2 u''(Y_f - p Q) + v''(Q)] \quad (P5.7)$$

As long as $\lambda v''(Q) \to 0$, (P5.7) will not hold. When $\lambda v''(Q) \neq 0$, it will generally not hold, even though for a small interval, it is possible that (P5.7) holds.

8.6 Proof of Proposition 6:
It suffices to derive the comparative statics only for a change in $Y_f$ which is also the comparative statistic of interest for the propositions that follow.
If $Y_m \leq Y_m \in (0, Y_f)$ thus $s = 0$, so the value of $Q$ is obtained from (14)

$$v'(Q) - pu'(Y_f - p Q) \leq 0 \quad (P6.1)$$

Differentiating (P6.1) and $f$'s budget constraint with respect to $Y_f$ yields the results stated in the proposition. Note that neither $x_m$ nor $s$ change with $Y_f$. In particular,

$$\frac{\partial Q}{\partial Y_f} = \frac{pu''(x_f)}{v''(Q) + p^2 u''(x_f)} > 0 \quad (P6.2)$$

$$\frac{\partial x_f}{\partial Y_f} = \frac{v'(Q)}{v''(Q) + p^2 u''(x_f)} > 0 \quad (P6.3)$$

8.7 Proof of Proposition 7:
If $Y_m > Y_m$, thus $s, Q > 0$.
Solving (17) and (18) for $\lambda$ and substituting in, yields the following system for $s$ and $Q$: 32
Totally differentiating the system in (P7.1):

\[
\frac{dQ}{dY_f} = \left[ \frac{p^2u''(x_m)u''(x_f)^2 + pu''(x_m)u''(x_f)v''(x_f)}{D} \right]^{dY_f} \]

Let \( D \) denote determinant of the Hessian which is equal to:

\[
D = \det \left[ \begin{array}{ccc}
-p^2u''(x_m)u''(x_f) + pu''(x_m)u''(x_f) & p^2u''(x_m)u''(x_f) - pu''(x_m)u''(x_f) \\
p^2u''(x_m)u''(x_f) - pu''(x_m)u''(x_f) & pu''(x_m)u''(x_f) - pu''(x_m)u''(x_f)
\end{array} \right]
\]

So, the comparative statics are,

\[
\frac{\partial Q}{\partial Y_f} = \frac{p^2u''(x_m)u''(x_f)^2 + pu''(x_m)u''(x_f)v''(x_f)}{D} > 0 \quad (P7.3)
\]

\[
\frac{\partial Q}{\partial Y_m} = \frac{p^2u''(x_m)u''(x_f)^2 + pu''(x_m)u''(x_f)v''(x_f)}{D} > 0 \quad (P7.3)
\]

\[
\frac{\partial x_m}{\partial Y_f} = \frac{pu''(x_m)u''(x_f)v''(x_f) - p^2v''(Q)u''(x_f)v''(Q)}{D} > 0 \quad (P7.6)
\]

\[
\frac{\partial x_m}{\partial Y_m} = \frac{pu''(x_m)u''(x_f)v''(x_f) - p^2v''(Q)u''(x_f)v''(Q)}{D} > 0 \quad (P7.6)
\]

Recall from FOC’s: \( v'(Q) - pu'(x_m) = \lambda v''(Q) \)

### 8.8 Proof of Proposition 8:

**Assumptions:**

(i) Spouse \( m \) can observe the transfer with probability zero.

(ii) The household public good is perfectly observable such that spouse \( m \) can infer that a transfer has occurred and thus adjust his contribution accordingly.

(iii) Spouse \( f \)'s private consumption, or discretionary expenditure, is not monitored by \( m \).

If \( f \) chooses to reveal the transfer and \( Y_m > \bar{Y}_m \) the demands are obtained from solving the following system of equations:

\[
u'(Y_m - s)[p^2u''(Y_f + s - pQ) + v''(Q)] - pv'(Q)u''(Y_f + s - pQ) = 0 \quad (P8.1)
\]

\[
pu'(Y_f + s - pQ) - v'(Q) = 0
\]

Thus, if \( f \) receives a transfer \( T \) and decides to reveal it, the change in \( Q, s, x_f, x_m \) per unit change in \( T \) are equivalent to those corresponding to changes in \( Y_f \) described in proposition 7.

The change in utility per unit change in the transfer is given by:

\[
\frac{\partial v'(Q)}{\partial T} = \frac{\partial v'(Q)}{\partial Y_f} \frac{\partial Y_f}{\partial T} + \frac{\partial v'(Q)}{\partial x_f} \frac{\partial x_f}{\partial T} = \frac{v''(Q)}{D} \left[ p^2u''(x_m)u''(x_f)^2 + pu''(x_m)u''(x_f)v''(x_f) \right]
\]

Substituting in \( f \)'s FOC \( pu'(x_f) = v'(Q) \),
If \( f \) decides to hide the transfer then \( f \) spends all the transfer on private consumption and the household good allocation doesn’t change compared to before the transfer, nor do \( m \)’s allocations. So it must be that \( u(x_f) < u(\bar{x}_f) < u(\bar{x}_f) \) where \( \bar{x}_f = x_f + T \) where \( x_f \) is the pre-transfer private consumption optimal allocation and \( \bar{x}_f \) is the post-transfer private consumption optimal allocation if the transfer is revealed. Thus, the change in utility per unit change in the transfer is give by:

\[
\frac{\partial u_f}{\partial T} \bigg|_{NR} = u'(\bar{x}_f) \tag{P8.3}
\]

Spouse \( f \) hides money from \( m \) if and only if

\[
\frac{\partial u_f}{\partial T} \bigg|_R = u'(\bar{x}_f) > 0
\]

Multiplying through by \( D<0 \),

\[
u'(\bar{x}_f)[p^4u''(x_m)u''(x_f)^2 + p^2u''(x_m)u''(x_f)v''(Q) + u''(x_m)v''(Q)^2 + p^2u''(x_m)u''(x_f)v''(Q)] > u'(\bar{x}_f)\left[p^2u''(x_f) + v''(Q)^2 + pv''Qv''Q + pu'''x''f\nu''Q + pu'''x''f\nu''Qp + 2v''Q\nu''Q\right]
\]

Which simplifies to,

\[
[u'(\bar{x}_f) - u'(\bar{x}_f)]u''(x_m)[p^2u''(x_f) + v''(Q)]^2 < u'(\bar{x}_f)\left[pv''(Q)u''(x_f)[v'(Q) - pu'(x_m)] + p^2v''(Q)u''(x_f)^2 - pu'''x''f\nu''Q\right] \tag{P8.6}
\]

Recall from (P7.6) that,

\[
\frac{\partial s}{\partial y_f} < 0 \quad \text{if} \quad pu'(x_m)u''(x_f)v''(Q) > pv''(Q)u''(x_f)[v'(Q) - pu'(x_m)] + p^2v''(Q)u''(x_f)^2
\]

So, when \( \frac{\partial s}{\partial y_f} > 0 \) (P8.6) doesn’t hold because left-hand-side is negative and right-hand-side is positive, so \( f \) never hides the transfer. However, when \( \frac{\partial s}{\partial y_f} < 0 \) both sides of the equation are negative, and the decision to reveal depends on relative preferences and the size of the transfer.

Consider the extreme case where \( f \) doesn’t hide and allocates all of the transfer towards the household public good, such that \( T = pQ \). As the transfer increases \( (T \to \infty) \), \( \lim_{T \to \infty} u'(\bar{x}_f) = \lim_{T \to \infty} u'(Y_f + T + s - pQ) = \lim_{T \to \infty} u'(Y_f + T + s - T) = u'(Y_f + s) \). If she does hide, then her only option is to allocate it towards private consumption to avoid detection, thus \( \lim_{T \to \infty} u'(\bar{x}_f) = \lim_{T \to \infty} u'(x_f + T) = u'(\infty) \to 0 \). The right-hand side of (P8.6) is negative and the left-hand side tends to zero, so in this case the equilibrium would be not to hide.

Now consider the other extreme case where the transfer tends to zero. If \( f \) reveals the transfer:

\[
\lim_{T \to 0} u'(\bar{x}_f) = \lim_{T \to 0} u'(Y_f + T + s - pQ) = u'(x_f), \quad \text{if she hides it} \quad \lim_{T \to 0} u'(\bar{x}_f) = \lim_{T \to 0} u'(x_f + T) = u'(x_f), \quad \text{so (P8.6) simplifies to} \quad 0 > u'(x_f), \quad \text{which always holds. Thus there exists a threshold level of transfer \( \bar{T} \) such that for any \( T < \bar{T} \) the Subgame Perfect Nash Equilibrium is to hide.}
\]

### 8.9 Proof of Proposition 9:

Totally differentiating the equations in (27) yields the following system of equations:

\[
\begin{bmatrix}
u''(x_f) + (1 - \mu)u''(x_m) & p(1 - \mu)u''(x_m) \\
p(1 - \mu)u''(x_m) & v''(Q) + p^2(1 - \mu)u''(x_m)
\end{bmatrix}\begin{bmatrix}dx_f \\ dq_f \end{bmatrix} = \begin{bmatrix}(1 - \mu)u''(x_m) & -u'(x_f) - u'(x_m) \\
p(1 - \mu)u''(x_m) & -pu''(x_m)
\end{bmatrix}\begin{bmatrix}dy \\ d\mu \end{bmatrix}
\]

Let the determinant of the Hessian be denoted by \( D \), where
$D = p^* \mu (1 - \mu) u''(x_f) u''(x_m) + \mu v''(Q) u''(x_f) + (1 - \mu) v''(Q) u''(x_m) > 0 \quad (P9.1)$

Comparative statics reveal that,

$\frac{\partial Q}{\partial y} = \frac{p \mu (1 - \mu) u''(x_f) u''(x_m)}{D} > 0 \quad (P9.2)$

$\frac{\partial x_f}{\partial y} = \frac{(1 - \mu) v''(Q) u''(x_m)}{D} > 0 \quad (P9.3)$

$\frac{\partial x_f}{\partial \mu} = -\frac{u''(x_f) + p(1 - \mu) u''(x_f) u''(x_m) + u''(x_m) v''(Q)}{D} > 0 \quad (P9.4)$

$\frac{\partial Q}{\partial \mu} = -\frac{p \mu u''(x_f) u''(x_m) + (1 - \mu) u''(x_m)}{D} < 0 \quad \text{if} \quad \mu u''(x_f) u''(x_m) > (1 - \mu) u''(x_m) u''(x_f) \quad (P9.5)$

$\frac{\partial x_m}{\partial y} = \frac{\mu v''(Q) u''(x_f)}{D} > 0 \quad (P9.6)$

$\frac{\partial x_m}{\partial \mu} = -\frac{p \mu (1 - \mu) u''(x_f) u''(x_m) + (1 - \mu) v''(Q) u''(x_m)}{D} < 0 \quad (P9.7)$

8.10 Proof of Proposition 10:

Assumptions:

(i) Spouse $m$ can observe the transfer with probability zero.

(ii) The decisions to reveal $T$ yields equivalent equilibrium allocations compared to the case of collective bargaining under perfect information.

(iii) If revealed, the transfer changes bargaining power.

(iv) Spouse $f$’s private consumption is not monitored by spouse $m$.

If $f$ chooses to reveal the transfer the demands are obtained by solving (26) for $T > 0$: Thus, if $f$ receives a transfer $T$ and decides to reveal it, the change in $Q, x_f, x_m$ per unit change in $T$ are equivalent to those corresponding to changes in $Y_f$ described in Proposition 9. The change in utility per unit change in the transfer is given by:

$\frac{\partial u_f}{\partial T} \bigg|_R = \frac{\partial y}{\partial Q} \frac{\partial Q}{\partial \mu} + \frac{\partial y}{\partial x_f} \frac{\partial x_f}{\partial \mu} + \frac{\partial y}{\partial x_m} \frac{\partial x_m}{\partial \mu} \quad (P11.1)$

$\frac{\partial u_f}{\partial T} \bigg|_R = \frac{v''(Q)}{D} \left( p \mu (1 - \mu) u''(x_f) u''(x_m) + \left[ p(1 - \mu) u''(x_f) u''(x_m) - \mu u'(x_f) u''(x_f) \right] \frac{\partial \mu}{\partial T} \right) + \frac{u'(x_f)}{D} \left( (1 - \mu) v''(Q) u''(x_m) - \left[ u'(x_f) v''(Q) + u'(x_f) v''(Q) + p^2 (1 - \mu) u'(x_f) u''(x_f) \right] \frac{\partial \mu}{\partial T} \right)$

Taking into account FOC’s $p \mu u'(x_f) = v''(Q)$,

$\frac{\partial u_f}{\partial T} \bigg|_R = \frac{p \mu u'(x_f)}{D} \left( p \mu (1 - \mu) u''(x_f) u''(x_m) + \left[ p(1 - \mu) u'(x_f) u''(x_m) - \mu u'(x_f) u''(x_f) \right] \frac{\partial \mu}{\partial T} \right) + \frac{u'(x_f)}{D} \left( (1 - \mu) v''(Q) u''(x_m) - \left[ u'(x_f) v''(Q) + u'(x_f) v''(Q) + p^2 (1 - \mu) u'(x_f) u''(x_f) \right] \frac{\partial \mu}{\partial T} \right)$

If $f$ hides then she allocates $T$ towards private consumption and neither household good consumption nor $m$’s private consumption change compared to before the transfer. So it must be that $u(x_f) < u(x_f) < u(x_f)$ where $x_f = x_f + T$ where $x_f$ is the pre-transfer private consumption optimal allocation and $\bar{x}_f$ is the post-transfer private consumption optimal allocation if the transfer is revealed. Thus, the change in utility per unit change in the transfer is given by:

$\frac{\partial u_f}{\partial T} \bigg|_{NR} = u'(x_f) \quad (P11.2)$

Spouse $f$ hides money from $m$ if and only if

$\frac{\partial u_f}{\partial T} \bigg|_R < \frac{\partial u_f}{\partial T} \bigg|_{NR} \quad (P11.3)$
Simplifying the above expression yields the condition that must be met for \( f \) to hide the transfer is given by

\[
\frac{\partial \mu}{\partial T} < \frac{1}{M} \left\{ u'(\bar{x}_f) \left[ p \mu v''(Q) u''(\bar{x}_f) + p^2 (1 - \mu) u''(\bar{x}_f) u''(\bar{x}_m) + (1 - \mu) v''(Q) u''(\bar{x}_m) \right] - p \mu^2 (1 - \mu) x_f u''(Q) u''(\bar{x}_m) \} \]

Where,

\[
M = p^2 \mu \left[ (1 - \mu) u''(\bar{x}_f) - \mu u'(\bar{x}_m) u''(\bar{x}_f) \right] - \left[ u'(\bar{x}_f) v''(Q) + u'(\bar{x}_f) u''(\bar{x}_m) v''(Q) + p21 - \mu x_f u''(Q) u''(\bar{x}_m) > 0 \right]
\]

\[
\lim_{\mu \to 0} M = -\left[ u'(\bar{x}_f) v''(Q) + u'(\bar{x}_f) u''(\bar{x}_m) v''(Q) \right] > 0 \text{ because } v'' < 0 \text{ and } u'' < 0 \text{ by assumption.}
\]

Whether there exists a strictly positive threshold change in bargaining power such that \( f \) hides is given by,

\[
u(\bar{x}_f) [p \mu v''(Q) u''(\bar{x}_f) + p^2 (1 - \mu) u''(\bar{x}_f) u''(\bar{x}_m) + (1 - \mu) v''(Q) u''(\bar{x}_m)] - \\
\mu (1 - \mu) v''(Q) u''(\bar{x}_m) - (1 - \mu) u'(\bar{x}_f) v''(Q) u''(\bar{x}_m) > 0
\]

Which simplifies to,

\[
u(\bar{x}_f) v''(Q) u''(\bar{x}_f) > \\
p^2 (1 - \mu) u''(\bar{x}_f) u''(\bar{x}_m) \left[ u'(\bar{x}_f) - u'(\bar{x}_m) \right] + (1 - \mu) v''(Q) u''(\bar{x}_m) \left[ u'(\bar{x}_f) - u'(\bar{x}_m) \right]
\]

\[
\lim_{\mu \to 0} M = -\left[ u'(\bar{x}_f) v''(Q) + u'(\bar{x}_f) u''(\bar{x}_m) v''(Q) \right] > 0 \text{ because } v'' < 0 \text{ and } u'' < 0 \text{ by assumption.}
\]

\[
\text{Whether there exists a strictly positive threshold change in bargaining power such that } f \text{ hides is given by,}
\]

\[
u(\bar{x}_f) [p \mu v''(Q) u''(\bar{x}_f) + p^2 (1 - \mu) u''(\bar{x}_f) u''(\bar{x}_m) + (1 - \mu) v''(Q) u''(\bar{x}_m)] - \\
\mu (1 - \mu) v''(Q) u''(\bar{x}_m) - (1 - \mu) u'(\bar{x}_f) v''(Q) u''(\bar{x}_m) > 0
\]

8.11 Proof of Corollary 1:

It follows from Proposition 10 that the change in bargaining power associated to income-hiding being the Subgame Perfect Nash Equilibrium must meet the following condition:

\[
\frac{\partial \mu}{\partial T} < \frac{1}{M} \left\{ u'(\bar{x}_f) \left[ p \mu v''(Q) u''(\bar{x}_f) + p^2 (1 - \mu) u''(\bar{x}_f) u''(\bar{x}_m) + (1 - \mu) v''(Q) u''(\bar{x}_m) \right] - p \mu^2 (1 - \mu) u'(\bar{x}_f) u''(\bar{x}_m) u''(\bar{x}_f) \right\}
\]

Where:

\[
M = p^2 \mu \left[ (1 - \mu) u''(\bar{x}_f) - \mu u'(\bar{x}_m) u''(\bar{x}_f) \right] - \left[ u'(\bar{x}_f) v''(Q) + u'(\bar{x}_f) u''(\bar{x}_m) v''(Q) + p21 - \mu x_f u''(Q) u''(\bar{x}_m) \right]
\]

Taking limit \( \mu \to 0 \) approaches zero:

\[
\lim_{\mu \to 0} \frac{\partial \mu}{\partial T} = \frac{[u'(\bar{x}_f) - u'(\bar{x}_m)] v''(Q) u''(\bar{x}_m)}{-u'(\bar{x}_f) v''(Q) - u'(\bar{x}_f) u''(\bar{x}_m) v''(Q) - p^2 u'(\bar{x}_f) u''(\bar{x}_m) v''(Q)} < 0
\]

Taking the limit as \( \mu \) approaches 1:

\[
\lim_{\mu \to 1} \frac{\partial \mu}{\partial T} = \frac{u'(\bar{x}_f) v''(Q) u''(\bar{x}_m)}{-p w(\bar{x}_f) u(\bar{x}_m) w(\bar{x}_f) - u'(\bar{x}_f) v''(Q) - u'(\bar{x}_f) u''(\bar{x}_m) v''(Q)} > 0.
\]

implying that the threshold change in bargaining power is increasing in the initial level of bargaining power.
References


37


